Proceeding of the 2007 Industrial Engineering Research Conference
G. Bayraksan, W. Lin, Y. Son, and R. Wysk, eds.

Sensitivity Analysis of a Real Options Problem

Neal A. Lewis
College of Information Technology and Engineering
Marshall University, South Charleston, West Virginia 25303, USA

Ted Eschenbach
TGE Consulting, Anchorage, Alaska 99504, USA

Joseph C. Hartman
Department of Industrial and Systems Engineering
Lehigh University, Bethlehem, Pennsylvania 18015, USA

Abstract

The existing literature describes how financial option techniques can be applied for determining a project option value. However, input variables are assumed to be fixed and known over the project's horizon. While this assumption may be accurate for short term financial options lasting a few weeks, it is rarely true for real options that last for years. An example problem is analyzed using both present worth and using real deferral options. Sensitivity analysis is conducted on both methods, showing how initial estimates and real world variability over the life of the project can impact the results of the valuation techniques.

Keywords
Real options, Present worth, Sensitivity analysis

1. An investment question
A drug company is seeking approval for a new drug product. If the company received approval from the Food and Drug Administration (FDA), the drug should be approved for sale two years from now. The drug will have patent protection for ten years after FDA approval (the 20-year patent was applied for 8 years ago at an earlier stage of the development process). Once on the market, year-one net cash from sales is expected to be $20M (million), year two cash flows are expected to be $28M, and years three through ten are expected to be $35M.

The facility to produce the new drug will take two years to build, and cost $38M with a $5M salvage value at the project horizon. There is a 90% chance that the FDA will approve the new drug. The hurdle rate for this project is 25%. For options calculations, volatility can be assumed to be 0.40. This value is typical for financial options for big pharmaceutical firms with volatility measured on an annual basis.

If facility construction begins after FDA approval, it will take two years to build the plant, and sales will be delayed two years. If facility construction begins now, it will be available to produce the drug if the FDA approves the drug. However, if the FDA does not approve the drug, the unused $35M facility will have a salvage value of only $9M at the end of year 2. This is the value of the facility if the FDA does not approve the project, rather than the terminal value at the end of the project.

The question facing the firm is whether the facility should be built now or should they wait until after FDA approval? The estimated base case values are summarized in Table 1 along with the lower and upper limits for each. Note that these limits are often asymmetric with the "bad" side often wider than the "good" side [1].

2. Traditional valuation
Discounted cash flow can be used to determine the present worth of the two alternatives, build now or build later. The two are mutually exclusive, and have the same anticipated time horizon.

The present worth (PW) to build now, adjusted for the probability of FDA approval, can be stated as:

$$PW_0 = -I_0 + P_{FDA}\sum_{t=1}^{n}\frac{CF_t}{(1+i)^t} + (1-P_{FDA})\frac{(\text{Unused salvage value})}{(1+i)} \quad (1)$$

where I_0 is the original investment
P_{FDA} is the probability of FDA approval of the project
CF_t are the future cash flows
i is the hurdle (interest) rate
t is the time increment

If the facility is built now, the investment, I_0, is not dependent on FDA approval. The production facility is completed in year two, in time for regulatory approval. Positive cash flow would occur in year three and continue for ten years until the patent expired. The alternative to build now has an expected present value of \$25.37M, given the values in Table 1. With FDA approval the PV is \$31.78M, and without it the PV is a loss of –\$32.24M. The standard deviation of the NPV is \$19.20M, and the P(loss) = 0.1.

The present worth to build later is calculated slightly differently, because the investment will not be made unless FDA approval is given. Equation 1 becomes

$$PW = P_{FDA}\left(-I_0 + \sum_{t=1}^{n}\frac{CF_t}{(1+i)^t}\right) \quad (2)$$

Table 1: Estimated values

Base Case		Lower Limit	Upper Limit
38	Investment (\$M)	-5%	15%
9	Salvage value unused yr 2 (\$M)	-100%	100%
5	Salvage value yr 12 (\$M)	-100%	100%
0.9	P(FDA approve)	-20%	10%
20	Revenue 1st year (\$M)	-40%	20%
28	Revenue 2nd year (\$M)	-40%	30%
35	Revenue >2 years (\$M)	-40%	40%
25%	*i - hurdle rate*	-20%	20%
5%	r_f *- risk free rate*	-40%	40%
2	N_{FDA} (yr)	-50%	50%
2	N_C construction (yr)	-50%	50%
10	N_P patent (yr)	-40%	20%
0.4	Volatility (yr)	-50%	50%

The investment would be made in year two if FDA approval is given. Positive cash flow would not occur until year 5 due to the delay in building the facility. Cash flows would occur for eight years instead of ten because patent exclusivity ends in year 12 as before. The alternative to build later has an expected present value of \$15.30M. With FDA approval the PW is \$17.00M and without it the NPV is \$0. Thus this alternative is less risky, since the PW's standard deviation is only \$5.10M, and the P(loss) = 0.

The decision can also be viewed as an option. There are two clear options available: build now or defer the construction to a point in the future when the FDA approval decision is known. The value of this inherent management flexibility can be determined through the use of real options. This is a deferral option.

The present worth analysis clearly signals that the facility should be built now, and the risk of the project not getting approved should be taken. The difficulty is that nearly all of the values are estimates, and very little can be stated with certainty. In order to improve confidence in the decision, option and risk analyses should be performed.

3. Options approach

An option is a right, without an obligation, to buy or sell an asset for a specified price at or before a specified time in the future. Fisher Black and Myron Scholes [2], along with Robert Merton [3] were the first to publish an accepted pricing model of financial options. The Black-Scholes model continues to be one of the most widely used methods of calculating option prices.

Real options analysis is based on the mathematics of financial options, and has been used to determine the value of projects. The basic Black-Scholes equation can be used to determine the option value of a simple deferral option. Cox, Ross, and Rubinstein [4] published the binomial option pricing method, demonstrating an alternative to the Black-Scholes model. The binomial model is a discrete time model, while Black-Scholes is a continuous time model. Binomial models use simple mathematics to price options and can be applied to a variety of options where exact formulas are not available. However, when dealing with very simple problems, such as the investment question here, the binomial model may not offer advantages.

There are key differences between financial options and real options. The outcome of financial option pricing is to identify an appropriate price of a financial security for the purpose of buying or selling. The outcome of real options analysis is to aid in decision-making, because options analysis provides a value for management flexibility (the ability to defer, abandon, expand, or contract a project) that may not be considered in typical discounted cash flow analysis. There is significant debate regarding the use of Black-Scholes for real option pricing. Some of the concerns include [5] [6]:

- Project volatility is not constant over time.
- There is no definitive expiration date of real options.
- The option may be exercised only at maturity.
- Real asset values and costs behave stochastically.
- Returns are not lognormally distributed.
- The random walk of real assets is not symmetric; there are jumps.
- There is only one source of uncertainty.

Additional concerns include:

- Real projects are not tradable assets, that is, there is no market. Thus, a replicating portfolio cannot be created.
- The connection between the project's uncertainty and the volatility is unclear.

Other concerns include the fact that forecasts used to determine the inputs tend to be far less accurate in real options than in financial options. The fact that the time period is no longer weeks (with financial options) but years (with real options) strains the ability to precisely forecast any of the inputs, and opens the doors to changes during the life of the option. Nevertheless, options analysis can provide a perspective that can be preferred over simple present worth analysis. The use of sensitivity analysis is needed to help compensate for some potential inaccuracies in the real options valuation methods.

In financial options, stock dividends often need to be considered. Dividends are received by the owner of the stock and not by the owner of an option. In real options, there is a corollary. While there is value in waiting for additional information, there is also a potential cost. There is potential lost revenue by not being in the market, other companies may market a similar product first, or there may be hidden costs of not funding the project. The deferral option needs to consider the cost of not making a decision.

The cost of waiting for the current investment problem may be determined by finding the PW of the incremental value between the "Build Now" and the "Build Later" alternatives. In our example, cash flows are different in years 3, 4, 5, and 6 of the projects. Discounting these differences using the hurdle rate provides a total cost of waiting, W, of \$28.46M.

The Black-Scholes equation approximates the value of a "European" call option (one that can not be sold before its maturity date), based on the current stock price (S_0), strike price (X), volatility (σ), risk-free interest rate (r_f) and time to expiration (t). In the Black-Scholes model, the dividend rate (D) can be accounted for as shown in equation 3 [7]. For the original Black-Scholes model, D = 0 and the exponential terms in Equations 3 and 4 equal 1.

$$C_0 = S_0 e^{-Dt}\Phi(d_1) - X e^{-r_f t}\Phi(d_2) \quad (3)$$

where $\Phi(d_x)$ is the cumulative standard normal distribution of the variable d_x
e^{-Dt} is the discounted cost of waiting, represented by a fraction

$$d_1 = \frac{\ln\left(S_0 e^{-Dt}/X\right) + \left(r_f + \sigma^2/2\right)t}{\sigma\sqrt{t}} \qquad d_2 = d_1 - \sigma\sqrt{t} \quad (4)$$

For real options, the financial variables translate to real variables, with the present worth of the future cash flows (S_0), the cost (X), project volatility (σ), risk-free interest rate (r_f), time to expiration (t), and the cost of waiting rate

(D). Because the total discounted cost of waiting has already been determined, the S_0e^{-Dt} term in both equations 3 and 4 can be replaced with (S_0-W).

Note that while the present worth calculation uses discrete compounding, the Black-Scholes equation uses continuous compounding. In present worth, the hurdle rate (*i*) is used, compounded annually where discounting is equal to $(1+i)^{-t}$. Black-Scholes uses the risk-free interest rate (r_f), and discounting is equal to $e^{-r_f t}$.

Unfortunately, the cost of waiting is usually dealt with in detail only in those books that treat real options in detail. Articles and book chapters may simply mention it or pass it by altogether. In reality, the cost of waiting can kill the option value of a real option. When dealing with any deferral option, the advantage of waiting needs to be offset by the cost of waiting.

The simple deferral option without the cost of waiting can be calculated using either the Black-Scholes model shown in equation 3 (with D = 0) or by using binomial lattices. The Black-Scholes model provides an option value of $29.3M; a binomial lattice will provide a similar answer, but the answer will vary depending on how many time steps are used. This option value exceeds the PW of the "Build Now" alternative, which implies that it is worth more to defer the project than it is to build. However, the cost of waiting is not included. Because there is a significant cost of waiting for FDA approval ($28.46M), it is important to include this. The option value including the cost of waiting is $7.98M, which is far smaller than either of the present worths. Because the largest value is the PW of the "Build Now" alternative, the best decision based on the predicted information is to build the facility now.

The inability to perfectly forecast future events will cause real option analysis to be less precise than financial option analysis. It is not worth refining an analysis technique to create a theoretically exact option price when the inputs are inherently inaccurate. All inputs may (and probably will) change during the course of the option period. Sensitivity analysis is needed.

4. Sensitivity Analysis

Sensitivity analysis is used to identify which variables (cash flows, time horizons, interest rates, etc.) are most influential on the outcome of the project. Sensitivity analysis can be used to concentrate our efforts and resources to make good estimates [8]. Three techniques were used to determine the sensitivity of the project's present worth and option value to the input variables: tornado diagrams, spiderplots, and Monte Carlo analysis. Due to the page limits the spiderplots are not presented here.

The range of the input variables is shown in Table 1. This shows the upper and lower limits of what we would expect regarding the accuracy of the estimates. For example, the investment is estimated at $38M, but the actual cost could be as much as 5% lower or 15% higher than the estimate. In other words, the investment is expected to be between $36.1 and $43.7M. This approach also applies to the other variables.

4.1 Tornado diagrams

Tornado diagrams are used to combine, contrast, and compare the relative sensitivity to the input variables. Tornado diagrams summarize the limits that these variables have on the present worth of the project. Figure 1 is a "stacked" tornado diagram that shows the sensitivities of both alternatives to the uncertainties in the data. The two "funnels" are created by ordering the variables from those with the most effect on the PW (placed at the top) to those with the least effect (placed at the bottom) [1].

The hurdle rate is clearly the most influential of the input variables. If the hurdle rate is at the low end of our range, the value of the project can double. However, if the hurdle rate is changed to the high end of the range, the project value can become quite small. The good news is that the hurdle rate is controlled within the organization, and while it is influenced by outside factors (such as the current cost of money), there is some internal level of control.

The ongoing revenue during years 3 through 12 is the next most important variable. It is common for project cash flows to be of extreme importance to the value of the project. Other variables can be seen in the diagram, each with decreasing influence to the value of the project. The volatility and the risk-free rate are not part of the present worth calculation.

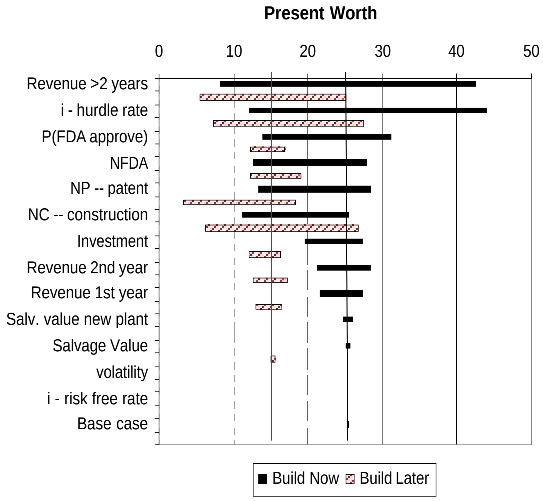

Figure 1: Tornado diagram of "build now" & "build later"

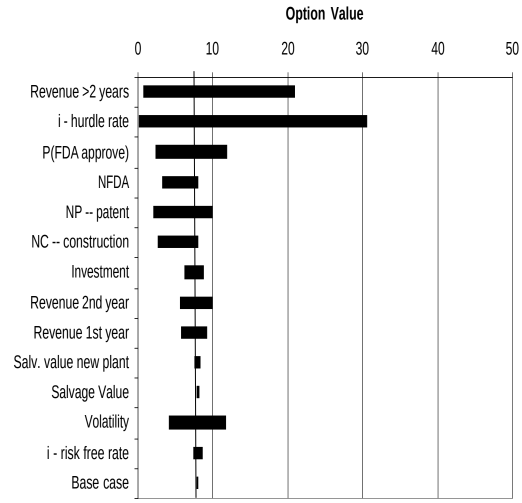

Figure 2: Tornado diagram of the option value

The tornado diagram for the "Build Later" alternative is also shown in Figure 1, and the two alternatives can be directly compared. Note that the level of influence of most of the variables is less pronounced in the "Build Later" alternative. The "Build Later" alternative has a lower average present worth.

The tornado diagram for the deferral option is shown in Figure 2. The largest driver of option value is the hurdle rate, which is also true for the present worth calculations. Ongoing revenue is the second most influential variable. All other variables have relatively small influences on the value of the option. By definition, a deferral option never goes negative. It is interesting to note that the same variables, with the exception of volatility, have the greatest influence over both the present worth and the option outcomes.

4.2 Monte Carlo

The present worth distribution can also be derived through simulation if the input parameter distributions are known. One approach is to use Monte Carlo simulation, which can be performed using one of several software packages, including Crystal Ball® and @Risk®. The resulting present worths are tabulated into a frequency histogram. For the simulation all variables (with the exception of the probability of FDA approval) were assumed to be triangular distributions with the same lower and upper limits as before, and the base case values used as the modes. The probability of FDA approval was set as a yes/no distribution, with "yes" set at 90% of the time.

The resulting present worth histogram for the "Build Now" alternative is shown in Figure 3. In this simulation, 5000 iterations were performed. This large number of iterations provides a reasonably reliable estimate of the present worth distribution. With all variables open to change at the same time, there is an 89% chance that the present worth will be positive. The histogram has a mean value of $18.46M (compared to a static value of $25.4M) and a standard deviation of 20.6. Note the bimodal distribution, due to the fact that PW is negative if the FDA does not approve the project.

The present worth histogram for the "Build Later" alternative is shown in Figure 4. Again, 5000 iterations were performed. With all variables open to change at the same time, there is a 98% chance that the present worth will be positive. The histogram has a mean value of $12.8M (compared to a static value of $15.3M) and a standard deviation of 9.1. If the FDA does not approve the project, the PW of the project is zero. It is interesting to note that the less risky alternative of building later has measurably better odds of delivering a positive present worth. While the value of the "Build Later" approach is lower, the chance of financial success is higher.

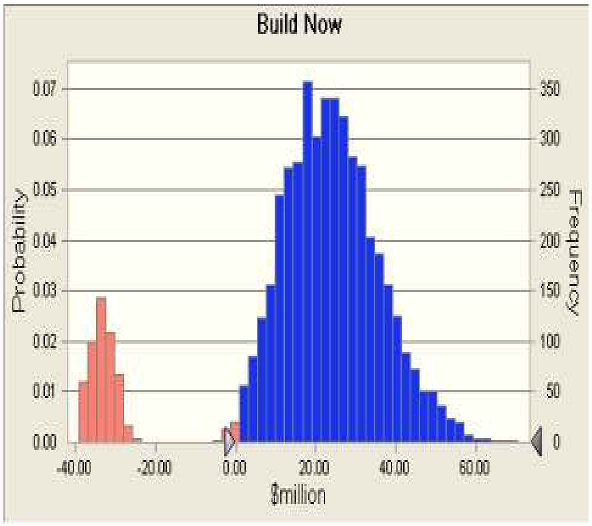

Figure 3: PW Histogram of “Build Now”

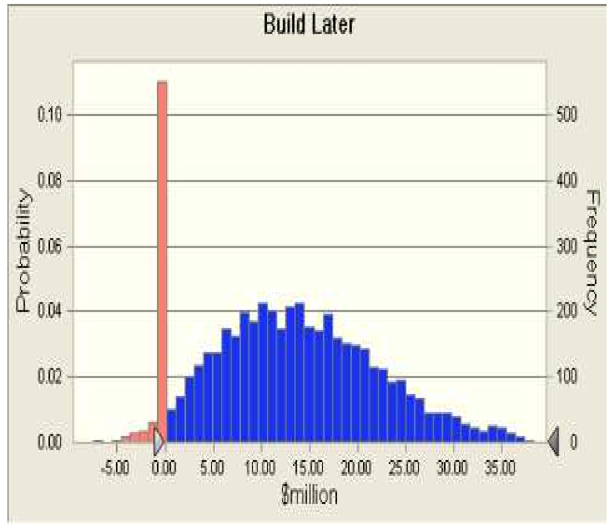

Figure 4: PW Histogram of "Build Later"

5. Conclusions

The most financially attractive alternative to the current problem is to build the manufacturing facility now. Traditional present worth analysis shows a positive present worth, and the PW is greater to build now rather than build later. Due to the high cost of waiting, the deferral option shows that the option value of waiting is far less than the present worth of building now.

While the deferral option can have its applications, it is important to include the cost of waiting. In the present case, the value of the option decreases dramatically when the cost of waiting is included. The deferral option that did not include the cost of waiting would have incorrectly appeared to be the preferred choice. The cost of waiting is a critical parameter for the deferral option, a fact that is overlooked in much of the real options literature.

The variables that drive the present worth also drive the option value, and the influence is in the same direction and largely to the same degree. For example, decreasing any of the net revenue values decreases the PW of each alternative and the value of the option. Project volatility, which is not included in the PW calculations, is an additional driver of option value. Varying the inputs will change the PW and the option value; however, these changes will generally not alter the final decision for the present problem.

References

1. Eschenbach, T.G., 2003, Engineering Economy, Applying Theory to Practice, 2nd edition, Oxford University Press, New York.
2. Black, F., and Scholes, M., 1973, “The Pricing of Options and Corporate Liabilities,” Journal of Political Economy, 81(3), 637-654.
3. Merton, R., 1973, “Theory of rational option pricing,” Bell Journal of Economics and Management, 4, 141-183.
4. Cox, J.C., Ross, S.A., and Rubinstein, M., 1979, “Option Pricing: A Simplified Approach,” Journal of Financial Economics, 7(3), 229-263.
5. Brach, M.A., 2003, Real Options in Practice, John Wiley & Sons, Hoboken, New Jersey.
6. Copeland, Tom, and Antikarov, Vladimir, 2001, Real Options, A Practitioner’s Guide, Texere, New York
7. Bodie, Z., Kane, A., and Marcus, A.J., 2002, Investments, 5th edition, McGraw-Hill Irwin, Boston.
8. Hartman, J.C., 2007, Engineering Economy and the Decision-Making Process, Pearson Prentice Hall, Upper Saddle River, New Jersey.