Abstract
Real Options Analysis is a technique that offers advantages over the traditional Discounted Cash Flow (DCF) approach for determining project valuation. Although options analysis uses some of the same input variables used in the DCF approach, it requires one additional variable, the volatility of the project’s forecasted returns, which is notoriously difficult to estimate reliably. There are several techniques that are used to model volatility when relevant historical data from similar projects are not available. This paper reviews the nature and potential limitations of these approaches, and provides recommendations regarding the appropriate uses of the estimates resulting from these methods.

Introduction
Projects are periodically evaluated to determine if they are feasible and worthy of continued funding. Most organizations have more ideas than they have resources to fund them, so projects compete for available resources, including money and talent. The most widely used technique for evaluating projects is discounted cash flow. In this method, the net present value (NPV) is determined by discounting forecasted future cash flows by a required rate of return. Despite its wide use, DCF suffers from a problem of being too conservative. Good ideas are sometimes not pursued because the method provides an NPV that is too low. The difficulty lies in the fact that management has flexibility during the course of development projects, and this flexibility is not accounted for in the DCF technique.

Projects with very high net present values are considered good investments from the DCF perspective. Projects with net present values that are negative are often abandoned because they do not appear to deliver the required return. Projects with net present values close to zero are dealt with in a variety of ways, and significant time is often spent trying to determine if such projects should be funded or abandoned. Real options analysis can be used to add insight to the funding decision, especially when DCF analysis finds a net present value that is close to zero. Real options analysis offers an alternative that determines a value for managerial flexibility and provides an expanded net present value (ENPV). In the case of funding under certainty, discounted cash flow analysis works very well. Under conditions of uncertainty, real options analysis may be preferred because project volatility is taken into consideration.

There are five primary management options regarding R&D projects. First, a project can be abandoned if its salvage value exceeds the project’s future returns at some time in the future. Second, a project may be delayed if future information will decrease the decision risks. Projects may be expanded or contracted at a later date, depending on market conditions. Finally, many projects occur in several phases, each phase dependent on the success of a previous one. Each of these scenarios represents distinct options that can be simulated. All of these options are dependent on five variables: the future cash flows, the cost of implementation, the time horizon under consideration, the risk-free cost of money, and the volatility of the future cash flows.

In discounted cash flow analysis, the required rate of return is adjusted to reflect the risk. Instead of using the weighted average cost of capital (WACC) as the interest rate, many firms increase the required rate to compensate for risk, creating what is known as the “hurdle rate”. High-risk projects carry higher required rates of return, which in turn decrease the NPV. The difficulty is that there is no direct relation between risk and the rate of return. Assignment of a hurdle rate must be arbitrary.

In real options analysis, rather than directly adjusting the required rate of return for the level of risk, the risk-free rate of return is used in conjunction with a separate volatility parameter. It is this inclusion of volatility that mathematically differentiates real options analysis from discounted cash flow. Options analysis recognizes that different types of projects will have different levels of volatility, or risk. Under real options analysis, when the volatility approaches zero, the valuation approaches the NPV. The volatility is the most difficult of all of the variables to forecast and measure, and the option value is highly dependent on the volatility estimate. A variety of methods can be used for estimating a project’s volatility.

Background
A financial option is an asset that gives the owner the right, without an obligation, to buy or sell another
asset (such as a quantity of corporate stock) for a specified price at or before some specified time in the future. An option that can be exercised only on its expiration date is known as a European option. An option that can be exercised at any time up to its expiration date is known as an American option. The option creates an expanded net present value (ENPV), which is defined as (Trigeorgis, 1996; Contractor, 2001):

\[ \text{ENPV} = \text{NPV} + \text{Option Value} \quad (1) \]

When NPV is quite large, the option value will not have a significant impact on the decision: the NPV signals that the project is worthy of investment. When NPV is very negative, even the best option values will not be large enough to create a positive ENPV, and the project should not be pursued. If the future cash flows are known with certainty, then the discounted cash flow technique should be used. Real options have their best use under conditions of uncertainty, and where management has the ability and the willingness to exercise its flexibility. The option value places a price on the worth of this flexibility, and the ENPV identifies how much the firm should be willing to pay to keep the project (or option) open.

**Call Options.** A call option gives the owner the right to buy an asset for a specified price, called a strike price or exercise price, on or before a specified expiration date. For example, an American call option on Microsoft stock with an exercise price of $27.50 and an expiration date of April gives the option’s owner the right to purchase Microsoft stock for $27.50 at any time until its expiration date in April. This call option flexibility will have a cost. The owner of the option is not required to exercise the option; there is no obligation to purchase the stock. If the price of the stock is greater than the strike price, then it may be profitable to exercise the option. If the stock price is above the strike price of a call option, the value of the call option will be positive and the option itself may be sold at a profit. If the stock price is below the strike price of a call option on the expiration date, there is no obligation to sell the stock and the option is allowed to expire. Because there is no obligation to sell the stock at a loss, the value of the call option never goes below zero.

**Put Options.** A put option gives the owner the right to sell an asset for a specified price on or before a specified expiration date. For example, an American put option on Microsoft stock with an exercise price of $27.50 and an expiration date of next April gives the option’s owner the right to sell Microsoft stock for $27.50 at any time until its expiration date in April. The owner of the option is not required to exercise the option; there is no obligation to sell the stock. If the price of the stock is less than the strike price, then it may be profitable to exercise the option. If the stock price is less than the strike price of a put option, the value of the put option will be positive and the option itself may be sold at a profit. If the stock price is above the strike price of the put option on the expiration date, there is no obligation to sell the stock and the option is allowed to expire. Because there is no obligation to sell the stock at a loss, the value of the put option never goes below zero.

**Valuation of Call and Put Options.** Fisher Black and Myron Scholes (1973), along with Robert Merton (1973), were the first to publish an accepted pricing model for options. The Black-Scholes model continues to be one of the most widely used methods of calculating option prices, despite many modifications that others have proposed over the years. The equation approximates the value of a European call option, based on the current stock price \( S_0 \), strike price \( X \), volatility \( \sigma \), risk-free interest rate \( r \), and time to expiration \( T \). The equation is:

\[
C = S_0 N(d_1) - X e^{-rT} N(d_2) \quad (2)
\]

where

\[
d_1 = \frac{\ln \frac{S_0}{X} + (r + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}}
\]

\[
d_2 = d_1 - \sigma \sqrt{T}
\]

\( N(d_1) \) is the cumulative standard normal distribution of the variable \( d_1 \).

There are limitations to the model, which were clearly stated in the original article. These include:

1. The short-term interest rate is known and is constant
2. The stock price follows a random walk in time and follows a lognormal distribution over time. The variance of the return on the stock is constant.

3. The stock pays no dividends.

4. The option is European, and can be exercised only at maturity.

5. There are no transaction costs in buying or selling the stock or option.

A derivation of the above equation is also available for determining the value of a put option, as based on the put-call parity theorem (Gibson, 1991).

\[ P = X e^{-rT} [1 - N(d_2)] - S [1 - N(d_1)] \]  \hspace{1cm} (3)

Cox, Ross, and Rubinstein published the binomial option pricing method, demonstrating an easier alternative to the Black-Scholes model. The binomial model is a discrete time model, while Black-Scholes is a continuous time model. Binomial models use simple mathematics to price options and can be applied to a variety of options where exact formulas are not available. Over the years, this method has received wide acclaim, and is the foundation of many tools that are used to value real options.

**Real Options.** Real options analysis is based on the mathematics of financial options, and has received widespread attention and acclaim since the early 1990s within academia. Very few companies have extensive experience with real options (Copeland, 2001). Examples of real options include licenses for oil exploration, an option to purchase electricity at a certain price at a future date, or an option to purchase land. Real options represent rights that are expected to be exercised later after more information becomes available about the value of that economic right. Real options therefore help in decision making under uncertain conditions. Judy Lewent, Chief Financial Officer of Merck, has said “When you make an initial investment in a research project, you are paying an entry fee for a right, but you are not obligated to continue that research at a later stage. Merck’s experience with R&D has given us a database of information that allows us to value the risk or volatility of our research projects, a key piece of information in option analysis. Therefore, if I use option theory to analyze that investment, I have a tool to examine uncertainty and to value it. … To me, all kinds of business decisions are options” (Nichols, 1994).

The five primary variables involved in the Black-Scholes calculation for financial assets can be directly related to real assets. These are shown in Exhibit 1 (Trigeorgis, 1996; Schweih, 1999).

**Exhibit 1. Option Variables.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Black-Scholes</th>
<th>Real Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Time to expiration</td>
<td>Time to expiration</td>
</tr>
<tr>
<td>r</td>
<td>Interest rate</td>
<td>Interest rate</td>
</tr>
<tr>
<td>X</td>
<td>Exercise price</td>
<td>Implementation cost</td>
</tr>
<tr>
<td>S</td>
<td>Stock price</td>
<td>PV of future cash flows</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Volatility of stock returns</td>
<td>Volatility of project returns</td>
</tr>
</tbody>
</table>

**Methods**

**Logarithmic Cash Flow Returns Approach.** Volatility must be estimated, whether we are dealing with financial options or real options. The Logarithmic Cash Flow Returns Approach calculates the volatility using either historic or future estimates of cash flows, along with their logarithmic returns. This method is widely used to estimate the volatility of financial assets. It assumes that the returns will follow a lognormal distribution. The logarithmic return of the cash flows is defined by

\[ r_i = \ln \left( \frac{S_i}{S_{i-1}} \right) \]  \hspace{1cm} (4)

which is an approximation of the percentage change (Rogers, 2002). This is best demonstrated through an example. Exhibit 2 illustrates an example of a project’s historic cash flows and their logarithmic returns.

**Exhibit 2. Example of the Volatility Calculation.**

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Cash Flow</th>
<th>Natural Log of Cash Flow Returns, ( r_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>(-0.223)</td>
</tr>
<tr>
<td>1</td>
<td>125</td>
<td>( \ln(125/100) = 0.223 )</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>( \ln(130/125) = 0.039 )</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>( \ln(120/130) = -0.083 )</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>( \ln(140/120) = 0.154 )</td>
</tr>
<tr>
<td>5</td>
<td>128</td>
<td>( \ln(128/140) = -0.090 )</td>
</tr>
</tbody>
</table>

The average log of the returns is \((0.223 + 0.039 - 0.083 + 0.154 - 0.090)/5 = 0.049\). The volatility estimate is then calculated as (Mun, 2002):
\[ \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (r_i - \overline{r})^2} \]  

The example problem is continued in Exhibit 3.

**Exhibit 3.** Example of the Volatility Calculation, Continued.

<table>
<thead>
<tr>
<th>Time</th>
<th>Natural Log of Cash Period</th>
<th>Flow Return, ( r_i )</th>
<th>( (r_i - \overline{r})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.223</td>
<td>0.0303</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.039</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.083</td>
<td>0.0174</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.154</td>
<td>0.0110</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.090</td>
<td>0.0193</td>
<td></td>
</tr>
<tr>
<td></td>
<td>average = 0.049</td>
<td>sum = 0.0781</td>
<td></td>
</tr>
</tbody>
</table>

The standard deviation of the returns is calculated as

\[ \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (r_i - \overline{r})^2} = \sqrt{\frac{1}{5-1} (0.0781)} = 0.140 \]

Because the original cash flows follow a lognormal distribution, the natural log of the returns will have a normal distribution. In this example, \( \sigma \) is equal to 0.140, or 14.0%.

This method is easy to implement, especially with the help of a computer spreadsheet. However, there is one primary problem that can limit its use. If any cash flow is negative during any time period, the method breaks down; the lognormal distribution assumption is invalid. If there is a negative cash flow, that time period cannot be calculated because the natural log of a negative number does not exist. If one time period cannot be calculated, then the volatility estimate will be inaccurate. The technique works well when analyzing stock prices (which follow a lognormal distribution), but may not be appropriate for real options analysis.

**Normal Cash Flow Returns.** Using our example from Exhibit 2, a normally distributed set of data will use a similar set of equations, as shown in Exhibit 4.

The standard deviation of the returns is calculated as

\[ \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (r_i - \overline{r})^2} = \sqrt{\frac{1}{5-1} (0.0902)} = 0.150 \]

In this example, \( \sigma \) is equal to 0.150, or 15.0%. This technique can be used without difficulty when individual data points are negative, which can occur when dealing with real projects.

**Monte Carlo Analysis.** Copeland and Antikarov (2001) explain a more detailed method of estimating volatility using Monte Carlo analysis. In this method, a spreadsheet is created, modeling the costs and revenues, and identifying the sources of uncertainty. Volatility is calculated using Monte Carlo simulations. In their example, they identify three potential sources of uncertainty: price per unit, quantity of output, and variable cost per unit. The project return \( z \) is then calculated, based on the results of the spreadsheet. Monte Carlo analysis can then be conducted using Crystal Ball® software.

The volatility of the project return will not be the same as the volatility of the inputs. The volatility of a gold mine will not be the same as the volatility of the price of gold, and the volatility of an electric utility stock will not be the same as the volatility of the price of coal or natural gas.

This technique can work very well if you know the variability of the inputs. The example assumes that you know how prices, quantities, and cost per unit will vary over a given time. The accuracy of the volatility estimate will only be as good as the estimates of the input variables (and their respective standard deviations). If the future will probably look like the past, then the use of historic data is an acceptable method. The alternative is to use forward-looking estimates.

**Management Estimates.** Management estimates of the future may also be used to determine volatility. By making “best case” and “worst case” estimates, volatility can be estimated. This best case/worst case method is widely used in industry today to predict product sales and financial budgets. The technique can be used to provide information for real options analysis with a minimum of effort. In addition to identifying the expected outcome for each variable, the responsible manager identifies a range of outcomes. Let us assume that the project has three sources of variability: price per unit, quantity,
and variable cost per unit. The manager would identify an expected outcome as before, but would also supply an optimistic forecast and a pessimistic forecast for these variables, using a 95 percent confidence.

The simplest, and probably least accurate method would be to create two spreadsheets, one for an optimistic and one for a pessimistic estimate. Two new estimates of the rate of return would then be identified. The volatility could be estimated by (Meredith & Mantel, 2003):

$$\sigma^2 = \left( \frac{b - a}{3.3} \right)^2$$  \hspace{1cm} (6)

where

- $b$ is the optimistic rate of return
- $a$ is the pessimistic rate of return
- 3.3 is equal to +/- $z$ at a 0.95 probability of a normal curve

In a more sophisticated method, these inputs can be applied to the original spreadsheet, not as hard volatilities, but as Monte Carlo inputs. The input variable is identified, and the mean, upper 95% limit, and the lower 95% limit are inserted. The return $z$ is identified as the output variable. The Monte Carlo analysis is performed, and a volatility estimate can be obtained as the standard deviation of the return $z$.

**Twin Security.** Trigeorgis (1996) recommends finding a traded security that is correlated with the project’s asset value, known as a twin security. In practice, finding such a security can be extremely difficult. Therefore, it is recommended that a twin security be selected that is highly correlated, and to use caution when interpreting the results. This approach can be used for three scenarios (Miller & Park, 2002):

1. Natural resource decisions because of the existence of a publicly traded commodity futures market
2. Firms evaluating a specific division within their company that find a traded stock of a company that mirrors their division’s value
3. When the project being evaluated contributes significantly to the firm’s market value, the company’s own stock is selected as the twin security.

For projects where an appropriate twin security can be identified in the market, the standard deviation of the historical returns of the twin security can be used as a proxy for the project’s volatility. Mun (2002) suggests that a common method of determining volatility is to use a company’s stock price. The company whose stock is being used should fit the definition of a twin security. A common problem with using stock price is that a firm’s stock prices are subject to investor overreaction and the psychology of the stock market, as well as the many other variables that may be included in the market that are totally irrelevant to a given project. Using an index that is made up of a number of these stocks would likely prove more useful as a twin security, rather than relying on a single stock.

**Implied Volatility.** In financial markets, the value of a publicly traded option is known and is available on listings such as the Chicago Board Options Exchange (CBOE). One common question is “What is the volatility that supports this price?” The implied volatility is the option market’s assessment of the expected future volatility of the underlying stock. This can be calculated using the Black Scholes pricing model or other models. Of course, the use of alternative models will give different results (this is known as model risk). Implied volatility can be an interesting way of comparing financial assets, because it is a statement of implied risk. For example, if a stock option has an implied volatility of 35 percent, this suggests that the shares have a two-thirds chance of trading within +/- 35 percent of the current stock price over the next year (Schaeffer, 1997).

Highly risky assets have high implied volatilities, and low risk assets have low volatilities. The S&P 500 Index has historically had an implied volatility assumption in the area of 12-15 percent, while some technology stocks have implied volatilities of more than 50 percent.

Within the Black-Scholes model, the underlying asset is assumed to be log-normally distributed and the volatility coefficient is constant. Because the model has been studied and used extensively over the last 30 years, it has been seen that historical and implied volatilities are not constant over time. According to Gesser and Poncet (1997), Black-Scholes based formulas tend to overprice at-the-money options and to underprice in-the-money and out-of-the-money options because of these assumptions.

**Results**

**Implied Volatility for Real Options Analysis.** In real options analysis, the volatility function is as important as it is in financial options. The key difference is that real options analysis uses estimates and forecasts to determine the future cash flows,
predicted costs, time horizons, and other variables. A real options analysis can never be as precise as a financial analysis because most of the inputs are forecasts, having their own inaccuracies. It is extremely difficult to accurately predict future sales volumes and income streams. Being able to do this while accurately estimating future volatility is even more difficult. Moreover, forecasters are usually not accustomed to thinking in terms of volatility, primarily because currently used tools (payback and discounted cash flow techniques) do not require this type of information. Real option analysis is by its very nature less precise; it is nearly impossible to develop an exact option value for a real project. However, using appropriate numbers, a reasonable estimate can be obtained that can be used to guide decision making.

The data can also be used for the relative comparison of projects. If there are doubts regarding the precision of the assumptions, then relative values (such as rankings) can generally provide acceptable results. The best use of real options analysis is to guide decision makers to choose the best course of action, not to provide an exact option price (Miller & Park, 2002). If the option analysis is used to compare projects having a similar risk, then the use of an estimate of volatility should give us acceptable results. In this context, the use of an appropriate twin security could be used as a proxy for a project’s volatility.

Twin securities were investigated to see if they could be used through their implied volatility. Several index options were studied to compare their volatility. Exhibit 5 identifies these index options, including some of the individual stock options that make up the index. Also included for comparison were options on several drug stocks. Data were collected from the Chicago Board Options Exchange website, www.cboe.com, during October and November 2003, and represent an average of weekly results gathered at week’s end during the period. The Black-Scholes pricing model was used to calculate the implied volatility from at-the-money call option prices.

As an example, the call option prices for Johnson & Johnson stock were identified on the CBOE website on October 26, 2003. The current price of the stock was $50.35. Exhibit 6 illustrates the ask prices for J&J call options. The maturity date for each of these options is the third Friday of the month. The current short term Treasury bill interest rate was 0.94%. For these three weeks, we have the stock price, the strike price, the option value, the time to maturity, and the risk-free interest rate. The implied volatility can therefore be calculated using the Black-Scholes equation (this is done iteratively). The implied volatilities are listed in Exhibit 7.

Projects that are close to at-the-money, that is, the present value of the cash inflows is nearly equal to the present value of the costs, are the projects in need of option analysis. Similarly, at-the-money implied volatility might be used as a surrogate for the project volatility, if there is an appropriate twin security.

Exhibit 5. Implied Volatility of Selected Index and Stock Options.

<table>
<thead>
<tr>
<th>Name</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow Jones Utility Index (DUX)</td>
<td>15%</td>
</tr>
<tr>
<td>Dominion Resources (D)</td>
<td>17</td>
</tr>
<tr>
<td>Exelon Corporation (EXC)</td>
<td>n/a</td>
</tr>
<tr>
<td>TXU Corporation (TXU)</td>
<td>29</td>
</tr>
<tr>
<td>Consolidated Edison (ED)</td>
<td>14</td>
</tr>
<tr>
<td>Public Service Enterprise (PEG)</td>
<td>19</td>
</tr>
<tr>
<td>GSTI Semiconductor Index (GSM)</td>
<td>39%</td>
</tr>
<tr>
<td>Texas Instruments (TXN)</td>
<td>40</td>
</tr>
<tr>
<td>Motorola (MOT)</td>
<td>43</td>
</tr>
<tr>
<td>Intel (INTC)</td>
<td>33</td>
</tr>
<tr>
<td>Applied Materials, Inc. (AMAT)</td>
<td>44</td>
</tr>
<tr>
<td>Morgan Stanley Biotech Index (MVB)</td>
<td>37%</td>
</tr>
<tr>
<td>Amgen (AMGN)</td>
<td>29</td>
</tr>
<tr>
<td>Biogen (BGEN)</td>
<td>35</td>
</tr>
<tr>
<td>Chiron (CHIR)</td>
<td>33</td>
</tr>
<tr>
<td>Genentech (DNA)</td>
<td>33</td>
</tr>
<tr>
<td>Genzyme (GENZ)</td>
<td>34</td>
</tr>
</tbody>
</table>

Drug Stocks

<table>
<thead>
<tr>
<th>Name</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merck (MRK)</td>
<td>26%</td>
</tr>
<tr>
<td>Pfizer (PFE)</td>
<td>24</td>
</tr>
<tr>
<td>Johnson &amp; Johnson (JNJ)</td>
<td>20</td>
</tr>
<tr>
<td>Bristol Myers Squibb (BMY)</td>
<td>43</td>
</tr>
<tr>
<td>CBOE Oil Index Options (OIX)</td>
<td>16%</td>
</tr>
<tr>
<td>Chevron Texaco (CVX)</td>
<td>18</td>
</tr>
<tr>
<td>Amerada Hess (AHC)</td>
<td>n/a</td>
</tr>
<tr>
<td>Total (TOT)</td>
<td>25</td>
</tr>
<tr>
<td>Conoco Phillips (COP)</td>
<td>n/a</td>
</tr>
<tr>
<td>Kerr-McGee (KMG)</td>
<td>20</td>
</tr>
</tbody>
</table>

It can be seen in Exhibit 5 that industries that would be expected to be of higher risk also have higher implied volatilities. The biotech industry in general has implied volatilities that are higher than the lower risk drug stocks. Likewise, companies that have higher volatilities than others within their industry
tend to be companies that are currently having problems. Note that Bristol Myers Squibb, which is currently having significant financial problems, has a much higher implied volatility than the other drug stocks that are of similar size. In the electric utility stocks, which are viewed as fairly low risk, TXU Corporation has a higher implied volatility than similar utility companies, implying that it is seen as a higher risk. On investigating this, it was found that TXU had a major drop in its stock price about one year before, and is still considered to be out of favor according to Standard and Poor’s. High liquidity, high volume stocks can also have higher than normal volatility due to day trading activities.

**Exhibit 6.** Johnson & Johnson Call Options, October 26, 2003.

<table>
<thead>
<tr>
<th>Maturity Date</th>
<th>Strike Price ($)</th>
<th>Time to Expiration (weeks)</th>
<th>Ask Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 03</td>
<td>50.00</td>
<td>4</td>
<td>1.05</td>
</tr>
<tr>
<td>December 03</td>
<td>50.00</td>
<td>8</td>
<td>1.75</td>
</tr>
<tr>
<td>January 04</td>
<td>50.00</td>
<td>12</td>
<td>2.15</td>
</tr>
</tbody>
</table>

**Exhibit 7.** Johnson & Johnson Implied Volatility.

<table>
<thead>
<tr>
<th>Maturity Date</th>
<th>Time to Expiration (weeks)</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 03</td>
<td>4</td>
<td>0.19</td>
</tr>
<tr>
<td>December 03</td>
<td>8</td>
<td>0.20</td>
</tr>
<tr>
<td>January 04</td>
<td>12</td>
<td>0.20</td>
</tr>
</tbody>
</table>

The option prices consider the future cash flows of the firms they represent, as well as the current and forecasted value of the companies. The implied volatility of the twin security makes a good proxy for real option volatility. Merck bases its project volatility estimates on a basket of biotech securities, and conducts options analysis at 40% volatility (Nichols, 1994). Note that the biotech index has an implied volatility of 37%, which is in line with this estimate. However, the grouping of many companies into an index (or a “basket”) will decrease the standard deviation relative to any individual company. The volatility of a single project would likely be higher than the volatility of a group. Merck also recalculates the option value using a volatility of 60% (Nichols, 1994), and estimates the ENPV to lie between these values.

Option prices increase with increased volatility, so a conservative options analysis will have a volatility that is not too high. It is more conservative to choose a low volatility than one that is too high. One could choose several values across a range for a sensitivity analysis, as will be demonstrated subsequently.

Amram and Kulatilaka (1999) recommend against inclusion of stochastic volatility in real options analysis. They explain that prices in commodity markets usually fluctuate around one stable level, and generally have a fairly constant volatility with only occasional spikes. Their experience is that most real options are virtually unaffected by unexpected changes in volatility. They claim that including stochastic volatility leads to more error in the real option results, and time is better spent on improving the accuracy of forecasts and identifying all of the possible options that may be imbedded in a project.

**Historical Analysis.** An historical stock price analysis was performed on the companies listed in Exhibit 5. These data are shown in Exhibit 8. Five years of data, from 1998 through 2002, were collected from publicly available information.

Many companies are leveraged; they use debt. Mun (2002) recommends that if the company has any significant debt, that the stock price volatility be modified as follows:

\[
\sigma_{RO} = \sigma_{EQUITY} \left(1 + \frac{D}{E}\right)
\]  

where

\(\sigma_{RO}\) is the volatility of the real option  
\(\sigma_{EQUITY}\) is the volatility of the stock price of the twin security  
D/E is the debt to equity ratio of the twin security

When debt is increased, a higher risk premium is generally required. Equation 7 will be used with the database, and will be identified as the “adjusted stock price”.

\[
\sigma_{\text{adjusted}} = \frac{\sigma_{\text{EQUITY}}}{1 + \frac{D}{E}}
\]
Exhibit 8. Stock and Option Volatility

<table>
<thead>
<tr>
<th>Implied Volatility</th>
<th>Stock Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short term</td>
<td>Stock Adj. Stock</td>
</tr>
<tr>
<td>S</td>
<td>S/(1+D/E)</td>
</tr>
</tbody>
</table>

**Semiconductors**
- GSM Index 39%
- TXN 43 65.7 58.0
- MOT 43 117.6 75.5
- ITC 33 44.7 43.1
- AMAT 44 73.4 64.1

**Oil**
- OIX Index 16%
- CVX 18 16.6 11.0
- AHC N/A 19.9 10.4
- TOT 25 16.4 11.4
- COP N/A 23.2 11.8
- KMG 20 32.6 13.3

**Biotech**
- MVB Index 37%
- AMGN 29 46.2 41.7
- BGEN 35 50.1 47.7
- CHIR 33 27.4 23.0
- DNA 33 40.9 36.0
- GENZ 34 60.1 46.9

**Drugs**
- MRK 26 32.6 20.0
- PFE 24 29.0 19.8
- JNJ 20 10.5 8.5
- BMY 43 52.5 36.5
- PHA 44.9 39.1

**Utility**
- DUX Index 15%
- EXC N/A 47.3 14.5
- TXU 29 50.8 14.5
- ED 14 6.1 3.1
- PEG 19 26.7 7.9

Copeland (1996), in his classic textbook *Valuation*, explains that the value of a business is the future expected cash flow discounted at a rate that reflects the risk of the cash flow. He goes on to say that the share price is a reflection of the value of the business. The stock market is not normally fooled by cosmetic earnings increases; the earnings increases that are associated with improved long-term cash flow will increase share prices (Copeland, 1996). There is evidence that accounting earnings are not well correlated with share prices. There is also evidence that the stock market evaluates management decisions based on their expected long-term cash flow impact, not their short-term earnings impact (Copeland, 1996). Given this, the stock price should be a good indication of long-term cash flow, and the volatility of the stock price should be a good indication of the risk related to that cash flow. These definitions are the type of proxy we are looking for in determining the volatility for real options. If we can find an appropriate twin security, then the adjusted stock price volatility should be a good surrogate for use in real options analysis. Similarly, stock options and index options are viable twin securities that could potentially be used to estimate project volatility.

There appears to be a good correlation between the volatility of the stock prices and the volatility of the options. If there is a good correlation, then there would be added support for using the implied volatility of the options as a proxy for project volatility.

Exhibit 9 shows the relationship between the stock price and the implied volatility using the short-term volatilities of all the firms. The correlation in Exhibit 9 is 0.830, and the coefficient of determination, $r^2$, is 0.690, or 69.0% of the variance in the option volatility is explained by the relationship. This relatively high correlation is statistically significant ($p < .01$).

Exhibit 10 shows the relationship between the adjusted stock price and the implied volatility, in the same way that Exhibit 9 was developed. The correlation in Exhibit 10 is 0.891, and the coefficient
of determination $r^2$ is 0.793. This correlation is also statistically significant ($p<.01$). The correlations are both logical and expected; the implied volatility of

Exhibit 10. Adjusted Historical Stock Volatility and Option Implied Volatility

![Graph showing adjusted historical stock volatility and option implied volatility](image)

Conclusions
Volatility is defined as the standard deviation of the investment’s return, whether this investment is a stock option or a project. Volatility can be modeled with the help of a computerized spreadsheet, or estimated using a variety of techniques. Monte Carlo methods can provide a sophisticated estimate of an asset’s volatility.

The use of a twin security to estimate volatility is widely described in the literature. These twin securities can be used to identify the implied volatility that can be used in the valuation of projects. A firm’s own stock can be used as a twin security if the project mimics the company’s average performance. It has been shown in this work that the implied volatility of a stock option or an index option can make an excellent forward-looking proxy for estimating the volatility of a project, assuming that an appropriate twin security is chosen. This approach can offer added insight to the investment decision, because risk can be quantified and related to the actual volatility of real firms.

References


About the Authors

**Neal Lewis** is a Lecturer in the Engineering Management Department at the University of Missouri – Rolla (UMR). He earned his B.S. in Chemical Engineering and Ph.D. in Engineering Management from UMR, and an MBA from the University of New Haven. He has over 25 years of industrial experience with Procter & Gamble and with Bayer Corporation.

**David Spurlock** is an Assistant Professor in Engineering Management at the University of Missouri - Rolla. He earned a Ph.D. in organizational psychology from the University of Illinois at Urbana-Champaign and has over ten years of engineering and management experience in industry. His research interests include individual and group judgment and decision making processes; managing people in organizations; organizational change, organizational development and program evaluation; and the influence of technological change on workplace behavior.