

Project Valuation for the Strategic Management of Research and Development: The Abandonment Option

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Abstract

The most widely used technique for evaluating projects is discounted cash flow. However, discounted cash flow analysis fails to consider flexibility. Real options analysis offers an alternative technique that provides value for the inherent managerial flexibility that most R&D projects contain. This paper investigates the abandonment option using computer simulation. There are five variables that determine the value of the abandonment option, and simulations analyze these variables over a wide range of conditions.

Introduction

Research and Development projects are periodically evaluated to determine if the projects are feasible and worthy of continued funding. Most R&D organizations have more ideas than they have resources to fund them, so projects compete for available resources, including money and talent. The most widely used technique for evaluating projects is discounted cash flow (DCF). In this method, the net present value (NPV) is determined by discounting forecasted future cash flows by a required rate of return. Despite its wide use, discounted cash flow suffers from a problem of being too conservative. Good ideas are sometimes not pursued because the method provides a net present value that is too low. The difficulty lies in the fact that management has flexibility during the course of development projects, and this flexibility is not accounted for in the discounted cash flow technique.

Projects with very high net present values are considered good investments from the DCF perspective. Projects with net present values that are negative are often abandoned because they do not appear to deliver the required return. Projects with net present values close to zero are dealt with in a variety of ways, and significant time is often spent trying to determine if such projects should be funded or abandoned. Real options analysis can be used to add insight to the funding decision, especially when DCF analysis finds a net present value that is close to zero. Real options analysis offers an alternative that determines a value for managerial flexibility and provides an expanded net present value (ENPV). NPV analysis is used to perform funding decisions for capital property. In the case of equipment purchases,

uncertainty is low because prices can be obtained ahead of time from the suppliers. In the case of funding under certainty, discounted cash flow analysis works very well. Under conditions of uncertainty, real options analysis may be preferred because volatility is taken into consideration. Under real options analysis, when the volatility approaches zero, the valuation approaches the NPV. Real options are an extension of discounted cash flow, not a substitution for it.

To date, real options have not received wide use within industry. This is mainly because real options are not widely understood by the managers who are responsible for the funding decisions. Managers simply will not take the large financial risk of funding a marginal project based on a technique that they don't understand.

The difficulty with the published literature lies in the fact that most publications do not address the topic for direct application. The early books and most of the journal articles focus on the mathematical derivations of the calculations, focusing on Ito calculus and differential equations. Most practitioners find this overwhelming. Some of the recent books deal with the mathematics very lightly or not at all, which fails to provide the industrial practitioners with the necessary tools. Even the best of the books (and there are excellent books available) are so long that a person needs to digest hundreds of pages before they are able to attempt a calculation. The topic of real options is complex, and the mathematics is cumbersome. Most of the published literature does not make the subject easy to apply.

There are four primary management options regarding R&D projects. First, a project can be abandoned if its salvage value exceeds the project's future returns at some time in the future. Second, a project may be delayed if future information will decrease the decision risks. Third, projects may sometimes be expanded at a later date if a product extension can increase market share. Finally, many projects occur in several phases, each phase dependent on the success of a previous one. Each of these scenarios represents distinct options that can be simulated. All of these options are dependent on five variables: the future cash flows, the cost of implementation, the time horizon under consideration,

the risk-free cost of money, and the volatility of the future cash flows.

The intent of this study is to investigate one method of valuating research and development projects using the abandonment option. The investigation identifies how to calculate an abandonment option, and identifies how the value of the option will change as the input conditions are varied over a wide range. The analysis will compare the relationships of future cash flow, investment costs, interest rates, time, and volatility with the estimated net present value of the project using computer simulations. Such valuation analysis can aid the firm in managing R&D projects for maximum strategic value.

Background

The valuation of a new business opportunity is dependent on both the knowledge of the firm and the business strategy and tactics that are used. Strategic use of intellectual capital provides the strengths for sustainable competitive advantage of the firm. A new opportunity will not see commercialization unless business strategy and tactics are taken into account. In evaluating research and development projects, two issues must be addressed: 1) how the new knowledge will bring value to the firm (strategically, not numerically), and 2) quantifying the amount of value that the asset will provide (Davis & Harrison, 2001).

Valuation is discussed extensively in academic literature and in the popular business press. The issue is relevant to accounting and finance, and valuation is a part of tax law. The strategic management of research and development is also widely discussed, especially in the business press. Numerous articles and books have been written on topics that include Intellectual Properties, Intellectual Capital, Intangibles, and similar topics, encouraging the maximum use of the products of R&D project work.

There are three accepted valuation methods used in accounting: market, cost and income (Smith & Parr, 2000). The income approach focuses on the income-producing capability of the project. Value can be defined as the present value of future benefits to be derived by the owner of the project. Therefore, the valuation process needs to quantify the future benefits, and discount them to their present value. In financial terms, the value of an asset can be measured by the present worth of the net economic benefit that can be achieved over the lifetime of the asset. For our purpose, the worth of the project is equal to what the project can earn. The income approach is the method that is best suited for assessing the value of an R&D project.

At the heart of the income approach is the discounted cash flow technique. This involves the determination of the Net Present Value (NPV) of future

cash flows by discounting the cash flows by a required rate of return. The discounted cash flow method is widely used to determine the value of projects, and has been embraced by industry. The required rate of return is typically the weighted average cost of capital of the firm (the interest rate that the firm must pay). However, if a firm wants to grow at a 20% rate, then it will want to fund projects that have this level of return or greater (known as a Hurdle Rate) (Meredith & Mantel, 2003). Although the discounted cash flow technique has its drawbacks, it is at the heart of all of valuation methods using the income approach.

Another method of determining the value of a project involves the use of real options. In general, the discounted cash flow method tends to be too conservative; good ideas are often not pursued because the method provides a net present value that is too low. The primary reason for this is the assumption that once the decision is made to fund a project, expenses and cash inflows occur without the possibility of being changed. In reality, management has options of making changes a number of times during the life of the project, especially during the early stages (Miller & Park, 2002).

Real options analysis has received widespread attention and acclaim since about 1995 within academia, but is just recently being applied widely within industry. Real Options Analysis is actually an extension of the DCF method (Brach, 2003). Very few companies have extensive experience with real options. However, one notable author feels that in ten years, real options will replace NPV as the central method for investment decisions (Copeland, 2001).

Copeland (2001) has developed a four-step process to describe the actions needed to properly carry out a real options analysis. The steps include:

1. Compute the base case present value without flexibility using standard Discounted Cash Flow valuation.
2. Model the uncertainty using event trees. This helps build an understanding of how the present value develops over time. This requires that the project be viewed strategically, identifying its risks and potential growth over time. A DCF analysis of the resulting event tree should yield the same result as in Step 1.
3. Identify and incorporate managerial flexibilities. By analyzing the event tree from Step 2, management options are identified, such as the option to abandon the project at a later date.
4. Conduct real options analysis. Once the event tree is identified with the known options, the computational analysis may be carried out. If uncertainty (σ) is zero, the present value is the same as in Step 1. If uncertainty is significant and

management has the ability to be flexible, the added option value can be significant.

Method

The first step in any option analysis is to identify the net present value, based on the equation

$$NPV = -I_0 + \sum_{T=1}^T \frac{FV_T}{(1+r)^T} \quad (1)$$

where I_0 is the original investment
 FV_T are the future cash flows
 r is the interest rate
 T is the time increment

Once the NPV has been calculated, the flexibility of a project can be determined.

An R&D project can be treated as an option. Management can choose to fund a project, abandon a project, delay a project, or expand a project. R&D projects can therefore be structured as real options. The value of the real option, and the value of the project in total, can be calculated in a similar way as financial options are calculated. There are two primary tools used: the binomial option pricing model and the Black-Scholes model. This work uses computer simulations to determine project values using both techniques. The binomial option pricing model is generally considered to be the more accurate technique, and is used here to map expanded net present values under a wide range of conditions.

A project must be structured with the real options identified. This requires that the project be viewed in a strategic context, with barriers and options highlighted. In the case of an abandonment option, the issue becomes one of identifying a salvage value opportunity. As an example, let us imagine a consumer products company that is developing a new product. The company is not yet sure that the product will be economically viable, and has been performing a financial feasibility study. The present value of the future cash flows has been estimated to be approximately \$10 million, but the volatility of the market is fairly high. The company has also found another firm that is interested in the new technology, and has identified that the project could sell all of its assets during the next five years for about \$8 million. The option to abandon the project and sell its assets has value. In discounted cash flow analysis, such a salvage value would be discounted to the present at the firm's working average cost of capital, with the assumption that the salvage value would be taken, and the cash flows past this time would cease. In real options analysis, the option to abandon the project and obtain the salvage value is simply an option; it is not an obligation. If the project is viewed as a European

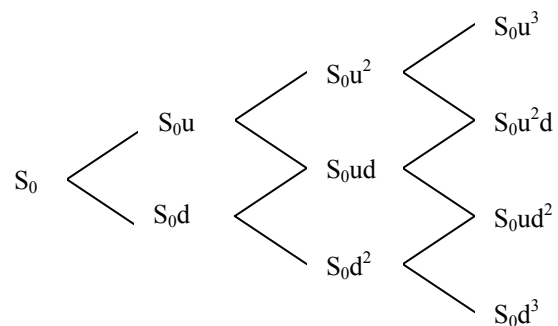
Option, the option could be exercised only at the end of the time frame. When valued as an American Option, it is assumed that the option can be exercised any time during the time frame (the next five years). The option creates an expanded net present value, which can be calculated:

$$ENPV = NPV + \text{Option Value} \quad (2)$$

where NPV is the same Net Present Value as in equation (1). When NPV is quite large, the option value will not have a significant impact on the decision: the NPV signals that the project is worthy of investment. When NPV is very negative, even the best of option values will not be large enough to value it as a profitable project, and the project should not be pursued. If the future cash flows are known with certainty, then the discounted cash flow technique should be used. Real options have their best use under conditions of uncertainty, and where management has the ability and the willingness to exercise its flexibility.

Binomial lattices. The binomial options approach uses a lattice to demonstrate alternative possibilities over time (Dixit & Pindyck, 1994). The lattice may be used for evaluating both real and financial options. The starting point is the present value of the future cash flows. Over time T , two conditions can result: one up and one down (hence the term binomial). More detailed lattices can be made to illustrate either more time or simply more steps in time. Exhibit 1 shows a binomial lattice with 3 time steps.

Exhibit 1. Binomial Lattice



The lattice solution can be obtained using one of two approaches. Financial options often use a market-replicating portfolio to solve the binomial problem. Real options can use a replicating portfolio, but generally use a risk-neutral probability approach. The two approaches are directly related and will yield the same answer if structured correctly. This paper works exclusively with risk-neutral probabilities.

Using the risk-neutral probability approach, each time-step may be calculated (Mun, 2002). The up-step is defined as

$$u = e^{\sigma \sqrt{\delta t}} \quad (3)$$

where σ is the volatility of the cash flows
 δt is the length of each time-step

The down-step is defined as

$$d = e^{-\sigma \sqrt{\delta t}} = 1/u \quad (4)$$

The risk-neutral probability is defined as

$$p = \frac{e^{r\delta t} - d}{u - d} \quad (5)$$

where r is the risk-free interest rate

Each type of real option requires a calculation in a slightly different way, but the solution always forms at least two lattices. This example demonstrates the valuation of an abandonment option. The first lattice, illustrated in Exhibit 2, shows the evolution of the underlying project. For our consumer product example above, let us assume the following:

- T = 3 years
- N = 3 time steps
- δt = 1 year (T/N)
- σ = 30% annual volatility
- r = 5 % (Treasury rate for a 3-year bond)

$$u = e^{\sigma \sqrt{\delta t}} = e^{(0.30)\sqrt{1}} = 1.35$$

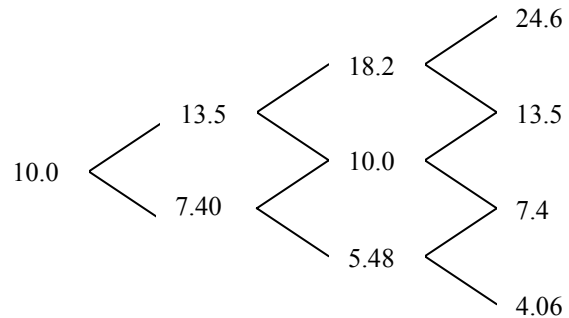
$$d = 1/u = 0.74$$

$$p = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{(0.05)(1)} - 0.74}{1.35 - 0.74} = 0.510$$

If the present value of the future cash flows, S_0 , is \$10 million, and $u = 1.35$, then the first 'up' position is S_0u or $(10)(1.35) = 13.5$. The down position is S_0d or $(10)(0.74) = 7.4$. This procedure is continued until the lattice is complete.

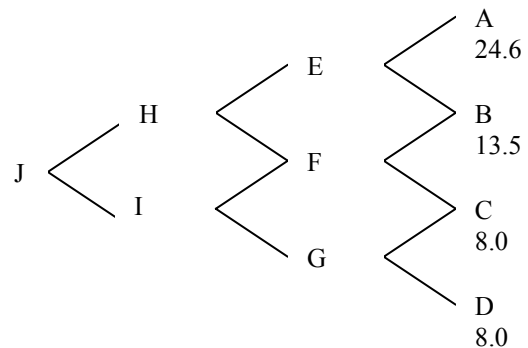
The second lattice, shown in Exhibit 3, is the option valuation lattice. Calculations start on the right side of the lattice, identifying the value of the option at that point in time. The ENPV at time T is first calculated at each node; it is either the evolved underlying asset or the salvage value, whichever is greater. In the top position A, the value is either S_0u^3 (which is equal to 24.6) or the salvage value (which is equal to 8.0 as stated above). Since the asset value is greater, the node value is 24.6. This same procedure is

Exhibit 2. Lattice of the underlying asset.



continued down the column, and Node B is valued at 13.5. At node C, the value is the greater of S_0ud^2 (7.40) or the salvage value (8.0). Since the salvage value is greater than the present value of future cash flows, it is worthwhile to exercise the abandonment option and to collect the salvage value. The node is valued at 8.0. Node D is valued at $\text{MAX}(S_0d^3, 8.0)$, so it also has a value equal to the salvage value of 8.0, and the project would again be abandoned.

Exhibit 3. Lattice of the Expanded NPV.



Internal points on the lattice are calculated using a method known as backward induction (Mun, 2002). The point is determined based on the probabilities of achieving the points already calculated on its right, discounted for the time period δt . The discounting is traditionally performed assuming continuous compounding. The equation for determining point E is then

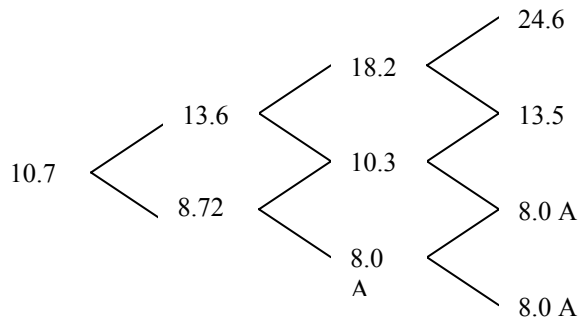
$$\begin{aligned} & [(P)(24.6) + (1-P)(13.5)] e^{-r\delta t} \\ &= [(0.51)(24.6) + (0.49)(13.5)] e^{-(0.05)(1)} = 18.23 \end{aligned}$$

This process is continued until the lattice is complete. At the extreme left side, the final value is the expanded

value of the discounted future cash flows. As shown in Exhibit 4, this value is 10.7. The option value itself can be determined by subtracting the original value, 10.0. The abandonment option is worth \$0.7 million.

The minimum value of any abandonment option is zero; options cannot be given a value less than zero. The theoretical maximum value of an abandonment option is its salvage value.

Exhibit 4. Complete option lattice.



While the binomial lattice can be easily understood once a person has a little experience with it, it is extremely cumbersome to calculate. Computer software is now available to calculate binomial lattices, which makes the procedure much faster. The Real Options Analysis Toolkit by Decisioneering, Inc. is used to calculate a variety of real option methods. This software is a spreadsheet-based (Excel) application that calculates real option values, expanded NPV, and identifies values at each point in a lattice. The Real Options Analysis Toolkit was used to perform the binomial lattice calculations used in this paper.

Black-Scholes. The Black-Scholes equation has been used for a number of years to determine the value of financial options (Bodie, 2002). Fischer Black, Myron Scholes, and Robert Merton won the 1972 Nobel Prize in Economics for this work. The equation approximates the value of a European call option, based on the current stock price (S_0), strike price (X), volatility (σ), risk free rate (r), and time to expiration (T). The equation is:

$$C = S_0 N(d_1) - X e^{-rT} N(d_2) \quad (6)$$

$$\text{where } d_1 = [(\ln S_0/X) + (r + \sigma^2/2)T] / \sigma \sqrt{T}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

and $N(d_x)$ is the cumulative standard normal distribution of the variable d_x

Manipulations of the Black-Scholes equations can become quite complex, taking the form of Ito calculus. For most applications, however, calculus is not necessary. A derivation of the above equation is also available for determining the value of a put option (Gibson, 1991).

$$P = X e^{-rT} [1 - N(d_2)] - S_0 [1 - N(d_1)] \quad (7)$$

The binomial lattice and the Black-Scholes equation will provide similar, but not identical, results. The binomial lattice assumes that the option can be exercised at any discrete time step (an American option). The Black-Scholes model is a continuous function, and assumes that the option can be exercised only at the expiration date (a European option).

The five primary variables involved in the Black-Scholes calculation for financial assets can be directly related to real assets. These are shown in Exhibit 5.

Exhibit 5. Option Variables

<u>Var.</u>	<u>Black-Scholes</u>	<u>Real Options</u>
T	Time to expiration	Time to expiration
r	Risk-free interest rate	Risk-free interest rate
X	Exercise price	Implementation cost
S	Stock price	PV of future cash flows
σ	Volatility of stock price movement	Volatility of future cash flows

(Schweih, 1999)

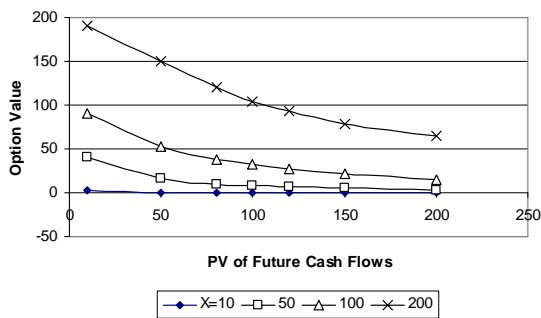
Sensitivity Analysis. Sensitivity analysis can be used in conjunction with real options. There are several sensitivity models widely used with financial options, known as the Greeks (Deacon & Faseruk, 1999). These are partial differential equations, derived from the Black-Scholes model. The Greeks define changes in option value relative to changes in each independent variable. For instance, delta is defined as the change in option value for each incremental change in the value of the underlying asset (S). Vega is the change in option value due to changes in volatility. Several of these tools are used extensively in tracking financial options, but there has been limited published research on their use with real options.

The Abandonment Option

Changes in the underlying asset. The value of the abandonment option will vary with the changes in the input parameters, and can be calculated using the computer software previously mentioned. One of the most critical parameters is the value of the underlying

asset. In the case of financial options, this is the price of the underlying stock. In the case of real options, this is the present value of the future cash flows of the project. Figure 6 shows the changes in the option value as the value of the underlying asset changes. The horizontal axis represents money, and can be considered as dollars or any other currency. The four separate lines represent four different salvage values, ranging from 10 to 200. For this graph, the volatility is held constant at 50%, the risk free interest rate is held constant at 5%, and the time frame is constant at 5 years with 5 time-steps. The shape of the curves follows those of the financial put options; the abandonment option is a form of a put option, and follows the same mathematics.

Exhibit 6. Option Value with Changes in Cash Flow; $\sigma = 0.50$, $r = 0.05$, $T = 5$.

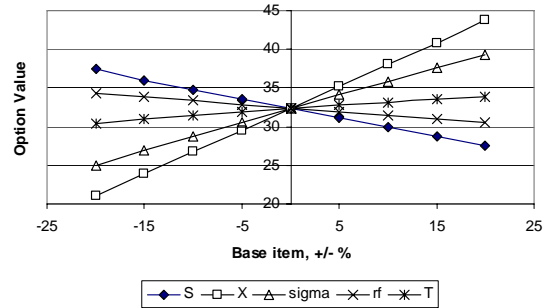


The maximum value of an abandonment option will be the salvage value X , just as a financial put option will have a maximum value equal to its strike price X . The minimum value is zero; the value will never be less than zero because the option can be allowed to expire without being exercised (you don't have to abandon the project).

Sensitivity. Numerous authors have described a standard sensitivity method, sometimes known as a Spiderplot, that compares a dependent variable to multiple attributes (Park, 1997; Eschenbach, 2003). This tool is a convenient way of showing the relative sensitivity to a number of variables. Exhibit 7 shows a spiderplot that demonstrates the sensitivity of the abandonment option value to the five dependent variables. The option value is most sensitive to changes in X , the salvage value, because this line has the greatest slope. In calculating the option value, estimates for the salvage value are the most crucial. The next most important variable is sigma, the volatility of the future cash flows, because this has the next largest slope. The least critical variable is the time horizon. The relative importance of each parameter is related to the magnitude of its slope. Interest rate and

time have negative slopes. This demonstrates that there is a negative correlation between these variables and the option value; as interest rate increases, the value of the option decreases. This is further explained below.

Exhibit 7. Sensitivity of the Option (center point at $S = 100$, $X = 100$, $\sigma = 0.50$, $r = 0.05$, $T = 5$)

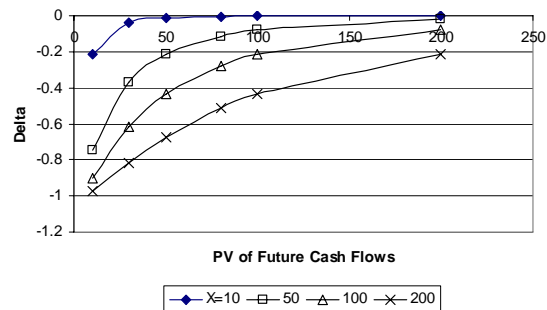


The sensitivity parameter for the project cash flows is known as Delta. It is defined as the change in option value for each unit change in the underlying asset S (the present value of future cash flows). For an abandonment option, Delta is defined as:

$$\text{Put Delta} = \partial P / \partial S = N(d_1) - 1 \quad (8)$$

This relationship is also known as the hedge ratio, and represents the slope of the curve at the given point. Exhibit 8 shows delta as calculated from equation (8). The value of Delta increases as the value of the underlying project increases, with a maximum value of

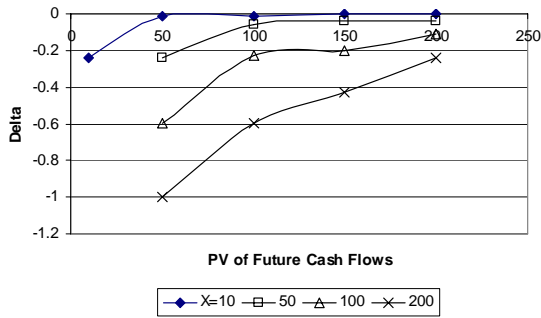
Exhibit 8. Put Delta based on partial differentials; $\sigma = 0.50$, $r = 0.05$, $T = 5$.



zero. The fact that delta is always negative confirms the inverse relationship between the option value and the future cash flows. The volatility, interest rate, and time are all held constant. An example of determining the sensitivity of the option value can be done using Exhibit 8. The value of delta is approximately -0.2

when $X = 50$ and $S = 50$. This means that for every unit increase in S , the option value will decrease by 0.2 under these conditions. Exhibit 9 shows delta as the discrete relation $\Delta P/\Delta S$, calculated using the binomial lattice. This graph is similar to $\partial P/\partial S$, shown in Exhibit 8.

Exhibit 9. Put Delta based on binomial lattice; $\sigma = .50$, $r = .05$, $T = 5$.



Salvage value. The accuracy of the option value is highly dependent on the accuracy of the stated salvage value. When forecasting, it is important to put initial emphasis on an accurate and reliable price that can be achieved if the project assets are sold. Exhibit 10 shows the nature of how the option value will change with changes in the salvage value X . The option value increases as X increases at all values of S , reaching a maximum value of $(X - S)$.

Exhibit 10. Salvage value; $\sigma = .50$, $r = .05$, $T = 5$.

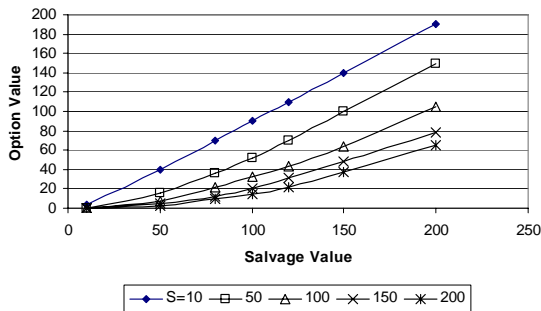


Exhibit 11 shows the sensitivity of the option value to changes in X . The change in the option value per unit change in the salvage value is sometimes referred to as the Greek Ξ , and is defined mathematically as the partial differential $\partial P/\partial X$. Exhibit 10 shows this relationship, based on the partial differential of the Black-Scholes equation:

$$\text{Put } \Xi = \partial P/\partial X = e^{-rT} [1 - N(d_2)] \quad (9)$$

Exhibit 11. Ξ , based on partial differentials; $\sigma = 0.50$, $r = 0.05$, $T = 5$.

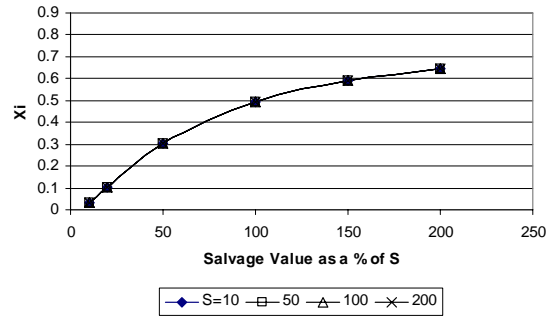
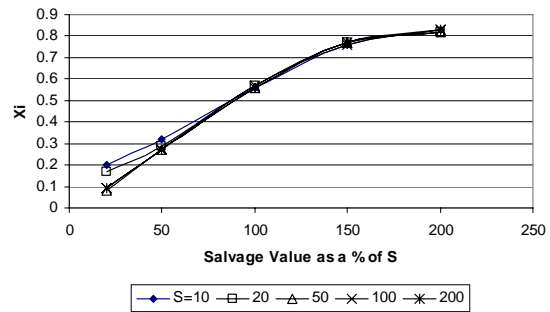


Exhibit 12 shows also shows this relation, but as the discrete relation $\Delta P/\Delta X$ where each ΔX is equal to one. The relationship is not dependent on S , the present value of the future cash flows, except at very small values of S and X . When both S and X are very small, a one unit change in X becomes significant, and the value of Ξ is proportionately greater.

Exhibit 12. Ξ , based on binomial lattice; $\sigma = 0.50$, $r = 0.05$, $T = 5$.



Comparing the two graphs, it can be seen that the relationship is very similar, with the binomial lattice being slightly greater. The option value from the binomial lattice is often larger than the Black-Scholes equation, since the Black-Scholes equation assumes a European Option (which can be exercised only at its maturity), while the lattice assumes an American Option (which can be exercised at any time up to its date of maturity). Hence, American options have a greater value.

Volatility. The second most important variable in estimating the option value is the variability of the future cash flows. Volatility is perhaps the most difficult of all of the variables to estimate, especially in an R&D scenario. The volatility is the standard deviation of the cash flows during the time period under question. The volatility must tie in with the

time-steps that are being used. For instance, if we are looking at a 5-year option with 5 time steps, then each time-step is one year. The volatility in question needs to be an annualized volatility. If one time step is one month, then the volatility must be the standard deviation of the monthly cash flows.

Merck has been using real options for several years, and has accumulated a large data base regarding the variability of their new initiatives (Nichols, 1994). Based on their internal database, Merck assigns a project a volatility of 40%, and repeats the analysis at a volatility of 60%. An analysis of Merck's cash flow over the past several years (based on published financial reports) shows annualized volatility of their corporate cash flow to be about 50%. This obvious relation comes as no surprise. Other industries tend to have widely different cash flows, with some industries (computer chips for example) having annualized volatility of 80% and more.

Exhibit 13. Volatility; $X = S$, $r = .05$, $T = 5$.

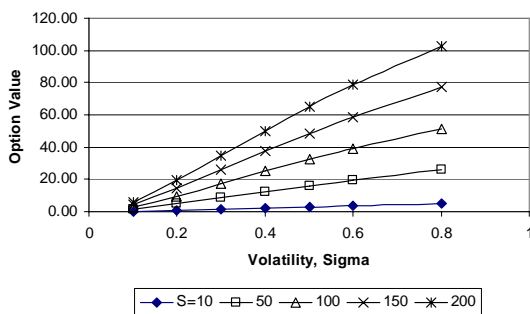


Exhibit 13 shows the relationship of the abandonment option value to changes in volatility when the salvage value X equals the asset value S . Option values increase with increases in volatility. This is because the probability of the upside potential increases as the variability increases. The probability of the downside potential does not increase, since the minimum value of the option is zero (Gibson, 1991).

The change in the option value per unit change in the volatility is known as the Greek term Vega. The graph shows that Vega reaches a maximum at about $\sigma = 30\%$. This same relation is true for both the discrete and the continuous calculation.

Exhibit 14 shows Vega based on the partial differential equation

$$\text{Vega} = \partial P / \partial \sigma = S \sqrt{T} N'(d_1) \quad (10)$$

$$\text{where } N'(d_1) = \exp(-\frac{1}{2}d_1^2) / \sqrt{2\pi} .$$

These equations also define the Call Vega, $\partial C / \partial \sigma$. Vega will always be greater than or equal to zero.

Exhibit 14. Vega, based on partial differentials, $X = S$, $r = 0.05$, $T = 5$.

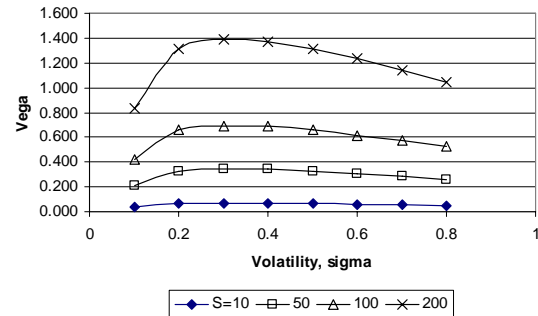
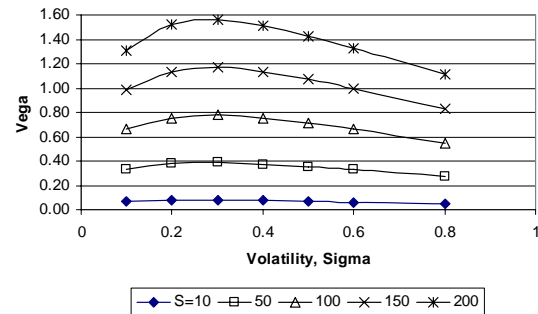


Exhibit 15 shows this relation as the discrete relation $\Delta P / \Delta \sigma$ where each $\Delta \sigma$ is equal to one percent. As in the calculation of X_i , the two forms of Vega are similar, with the discrete function having a value slightly higher.

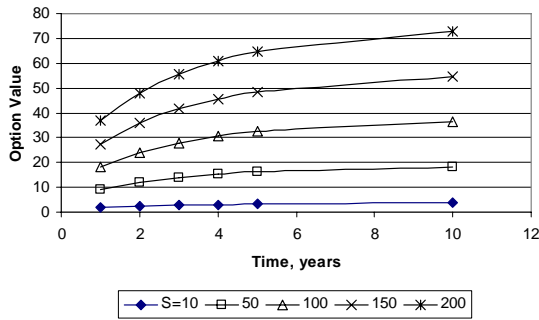
Exhibit 15. Vega, based on binomial lattice, $X = S$, $r = .05$, $T = 5$.



Time to maturity. The time variable is the time from the present until the time that the option might be exercised. If a project is being considered for abandonment sometime in the next five years, then the time to maturity is five years. The timeframe is important for the other factors as well, as it determines the timeline that the option is being valued. A five year timeline means that the interest rate must be a 5-year interest rate and the volatility must be based on an annualized standard deviation.

The abandonment option value increases the longer that the option is held open. The option increases in value because the chances of ending with a positive value increase with time, while the chances of ending with a negative value do not (the option will never be worth less than zero). The relationship is shown in Exhibit 16.

Exhibit 16. Time relation; $X = S$, $\sigma = 0.50$, $r = 0.05$.

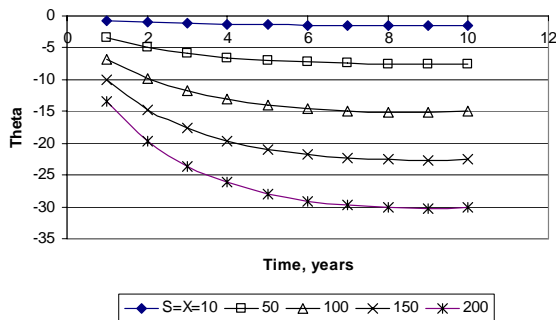


The sensitivity function for time is known as theta, defined by

$$\frac{\partial P}{\partial T} = (S\sigma/2\sqrt{T}) N'(d_1) + X e^{-rT} r [N(d_2) - 1] \quad (11)$$

The continuous function for theta, based on equation 11, is shown in Exhibit 17. Theta will generally be negative. The longer the time horizon, the longer the time the salvage value must be discounted, and the lower the resulting option value will be.

Exhibit 17. Theta, based on partial differentials; $X = S$, $\sigma = 0.50$, $r = 0.05$

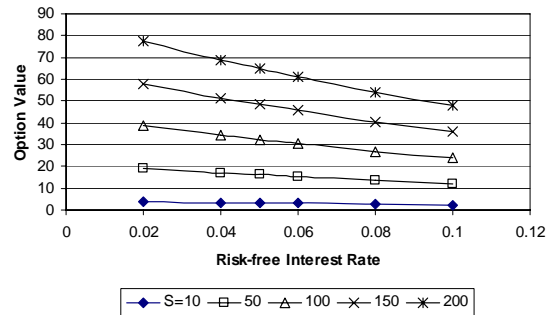


Interest Rate. The interest rate used in the abandonment real option is a risk-free interest rate. In discounted cash flow, the interest rate is often inflated to compensate for risk. "Hurdle rates" are often used instead of the Weighted Average Cost of Capital to ensure a high return and to hedge against risk. Unfortunately, risk and interest rates are difficult to correlate with any accuracy. In real options, risk is transferred to the volatility function, and is directly used in the calculation of the binomial lattice. The interest rate used in real options is therefore a risk-free rate based on the time horizon. If the project has a timeline of 5 years, then choose the rate for 5-year Treasury bonds. If the project has an option timeline

different from 5 years, use a corresponding Treasury rate.

The option value decreases with increasing interest rates, as shown in Exhibit 18. A higher interest rate makes the present value of the salvage option less valuable. This decreases the current exercise value ($Xe^{-rT} - S$) of the option (Gibson, 1991).

Exhibit 18. Risk-free rate; $X = S$, $\sigma = 0.50$, $T = 5$.

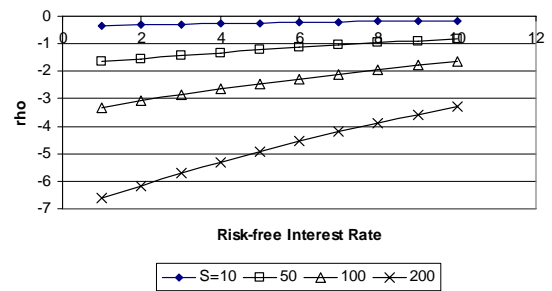


The sensitivity function for the interest rate is known as rho, and is defined by

$$\frac{\partial P}{\partial r} = TXe^{-rT} [N(d_2) - 1] \quad (12)$$

The continuous function for rho, is shown in Exhibit 19. Rho is always negative for an abandonment option, demonstrating the fact that the option price is negatively correlated with the interest rate. The positive slope of rho shows that incremental increases in the interest rate have less affect as the rate increases.

Exhibit 19. Rho based on partial differentials; $X = S$, $\sigma = 0.50$, $T = 5$.



Sensitivity to the interest rate is relatively minor compared to the affect of the salvage value or the volatility.

Conclusions

The abandonment option can provide for a more accurate valuation of a project. The value of this option is dependent on five variables:

- The present value of the sum of future cash flows
- The salvage value of the abandonment option
- The volatility of the future cash flows
- The timeline until the option is exercised
- The risk-free interest rate

Use of real options provides a value for management flexibility, and therefore more accurate project valuation.

Given our base condition (S=100, X=100, σ = 50%, r = 5%, and t = 5 years), a 20% increase in each of the variables will provide a change in the option value as follows:

Salvage value	+26.15 %
Volatility	+17.55 %
PV of future cash flows	-14.74 %
Timeline	+6.13 %
Interest rate	-4.26 %

The above sensitivities show the effect on option value as a single variable changes. In reality, there are interactions among the variables. Interest rates are based on the time horizon. Volatility must be based on the appropriate time increment (months or years). Interest rate increases decrease the option value because of the discounting affect on the salvage value. Many of the variables are interdependent.

The salvage value, volatility, and future cash flows should be forecasted with great care. The interest rate can be estimated with somewhat less precision.

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