Abstract
Managing natural resource projects requires that future costs and revenues be forecasted. Most commodity pricing models are fairly simple, involving a slow, steady increase in base prices while including volatility. When analyzing stock prices, this pattern is common referred to as the ‘random walk’. Complicating the forecasting process is the fact that many commodities exhibit mean reversion tendencies, where prices may fluctuate about a long-term mean. Stock price volatility is measured using the standard deviation of the return of a stock. Commodity price volatility is the same; however, mean reversion exists, the normal standard deviation will underestimate true volatility. This complicates the pricing of many types of derivatives that are based on commodity prices. This work investigates the mean reversion tendency of oil prices. Specifically, West Texas Intermediate daily oil prices is analyzed to determine price volatility, mean reversion speed, and the adjusted volatility that should be applied to today’s oil-related projects.

Introduction
Real options analysis is a tool intended to value management flexibility in future decisions. The mathematical foundation of real options is based on physics and engineering, stated as some variation of Equation (1), representing geometric Brownian motion (Hull, 2009).

\[ dS_t = \mu S_t dt + \sigma S_t dZ_t \]

where \( S_t \) is the stock or commodity price \( \mu \) is the expected rate of return \( \sigma \) is the volatility of the stock or commodity price \( dW_t \) is a Wiener process

While many stocks and some commodities are lognormally distributed with Brownian motion, most real projects are not.

Mean Reversion
It has been well documented that many commodities exhibit Brownian motion involving mean reversion. The underlying theory is that prices stay within a range, largely because investors are risk averse so prices are driven to their long-term means. Mean reversion is a concept that was first utilized by Sir John Hicks in 1927 (Hicks, 1927). Some commodities, including oil and natural gas, exhibit mean reversion along commodity price movement. Over time, prices revert to a long-term mean. Particularly high oil prices will, in the end, fall back to a long-term mean, and particularly low prices will rise over time. Schwartz (1997) presented several models that may be used to describe price movements in the presence of mean reversion. His Model 1, shown in Equation (2), is a one-factor model that assumes the logarithm of the spot commodity price follows a mean reverting process.

\[ dS_t = k(S_t - S_0) dt + \sigma dZ_t \]

where \( k \) is the speed of reversion

Halter (2003) demonstrated several models using approximations for use in determining the mean reversion speeds under stochastic volatility and mean reversion. This work included models of adjusting the volatility to take into account the mean reversion nature of a commodity. The resulting adjusted volatility could be used in standard models, including Black-Scholes.

Method
Historic spot prices of West Texas Intermediate crude oil are analyzed. The U.S. Department of Energy publishes daily and weekly spot prices for West Texas Intermediate crude oil on their website (energyinfo.net). The database begins in 1986 and continues to the present. This data has been studied for several topics:

• To verify that crude oil prices follow a lognormal distribution
• To determine the volatility of crude oil prices
• To determine the mean reversion speed of crude oil prices
• To determine the variables and constants that pertain to the commodity price model [Equation (2)].

Several studies have estimated the mean reversion speed by modeling futures prices (Schwartz, 1997) or using graphical approaches (Skorodumov, 2008) with different outcomes. Using option prices should be more accurate, and the current information provides a large historical database for graphs. Previous graphical approaches do not work well because volatility (scatter in the data) tends to hide short-term trends.

Results: Distribution
Oil prices are lognormally distributed, as identified in the literature. A histogram of the long-term database is shown in Figure 1, which verifies the lognormal distribution of WTI oil prices. The lognormal fit was verified using @Risk software.

Results: Volatility
Price volatility is the standard deviation of the price rate of return, where this rate of return is defined as Equation (3).

\[ \sigma = \sqrt{-\ln S_t} \]

where \( \sigma \) is the rate of return \( S_t \) is the current stock or commodity price \( S_0 \) is the stock or commodity price in time period 1

Because the database is comprised of daily prices, the resulting standard deviation over a period time will be a daily volatility. To convert this to an annual volatility, it must be multiplied by the square root of the number of trading days per year (252 on average), as shown in Equation (4).

\[ \sigma_{\text{annual}} = \sigma \times \sqrt{252} \]

Results: Mean Reversion
Oil prices exhibit mean reversion. The frequency of prices reverting to their long-term mean is not constant, and depends on the time period. Figures 3 and 4 show the oil price run chart along with the best fitting trend line (drawn using Excel). Figure 3 shows the trend for the past five years 2006 through 2011, with each mean reversion based simply on the graph. This graph shows 13 mean reversion periods over a period of 12 years, for a mean reversion speed, \( k \), of 15/12 years of 1 year⁻¹.

Figure 1. WTI Spot Oil Price, 1986 – 2012.

Figure 2. WTI Spot Oil price histogram, 1986 – 2011

Figure 3. Mean reversion of WTI Crude Oil, 2006-2011

Figure 4. Mean reversion of WTI Crude Oil, 2007-2011

Method, continued
Daily spot oil prices are used for the time period of 1986-2011. These prices are shown in Figure 1.

Results: Mean Reversion
Figure 4 shows the same information, but for the five-year period of 2007-2011. During this period, there were 7 mean reversion, for a mean reversion speed, \( k \), of 1.75 years or 1.4 year⁻¹. This compares with estimates made by Schwartz (1997) of 0.3 to 0.7 for other prices, and Al-Harthy’s (2007) estimate of 0.7. Clearly, the mean-reversion constant is not constant.

Adjusted Volatility
Halter (2003) demonstrated that volatility must be adjusted when mean reversion is present. He adjusted volatility by solving Equation (5).

\[ \sigma_{\text{annual}} = \sqrt{-\ln S_t} \times \sqrt{252} \]

where \( \sigma_{\text{annual}} \) = variance with mean reversion \( t \) = beginning time period, usually 0 \( T \) = time horizon \( \kappa \) = mean reversion speed \( \sigma \) = incremental/time, left as a variable \( \sigma_0 \) = standard deviation (volatility) without mean reversion

The effect of the variables \( \kappa \) and \( \sigma_0 \) are shown in Figure 5. In this graph, these two variables can be used to adjust volatility. For example, if an oil development lease were held for 2 years, a real deferral option having a time horizon of 2 years, with a mean reversion speed of 1.0 and an oil price volatility of 35%, the effective (adjusted) price volatility would be 0.173, half of the original volatility. Because options and other derivative prices are highly dependent on volatility, this would make a dramatic effect on the value of the option.

Conclusion
Crude oil prices follow Brownian motion with mean reversion, as identified in the literature. West Texas Intermediate crude oil follows a lognormal distribution with slowly changing volatility. The mean reversion speed is not constant, but has shown a cycle length of about 11 months over the past decade, and about 9 months over the past five years. The mean reversion speed is fast enough to have dramatic affects on the adjusted price volatility. The price volatility that should be used for hedging and derivative analysis is significantly lower than standard methods would indicate.

Many businesses delay oil and gas development on marginal projects until the price increases. When world oil prices increase, a huge number of marginal projects are funded. Unfortunately, most of these projects require several years to complete. Due to the mean reversion nature of the price, by the time the projects are completed, prices will have reverted to their long-term means. It is risky to depend on rising oil prices to justify the development of marginal oil development projects.

The standard methods of determining volatility will over-value derivatives, including real options, on commodities that are mean reverting. Adjusted volatility must be used that incorporate mean reversion to avoid overly optimistic derivative prices.

References

Table 1. Annual Price Volatility, WTI Crude Oil.

<table>
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<tr>
<th>Year</th>
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Figure 5. Adjusted Price Volatility, T=2.