Abstract: We solve two “unsolvable” (teyku) problems from the Talmud that had remained unsolved for about 1,500 years. The Talmudic problems concern the implied decision-making of farmers who have left some scattered fruit behind, and the alleged impossibility of knowing whether they would return for given amounts of fruit over given amounts of land area if we are aware of their behavior at exactly one point. We solve the problems by formalizing the Talmudic discussion and expressing five natural economic and mathematical assumptions.

THE TEYKUS

The Mishna records: "If he found scattered fruits" [the finder may keep them]. How much (fruit scattered over how much area)? R. Yitzhak said: One kav over an area of four amot.
R. Yirmiaḥ asked: What of half a kav over a two amah area. Is it the case that a kav spread over four amot belongs to the finder because the collection effort is too great (so that the owner abandons the stuff) -- so that, in the case of half a kav over a two amah area, which does not involve so much work, the owner does not abandon his rights and the finder cannot claim the grain? Or perhaps the rationale for the kav per four amot ruling is because a kav simply isn’t enough to be concerned with -- in which case, a half kav is certainly insignificant, and the owner does renounce ownership (and the finder can claim it)?
R. Yirmiaḥ’s second question: Two kavs over an eight amah area -- what is the law? Is the kav per four amot rule because the collection effort is too great, in which case the collection effort in an eight amah area is certainly excessive, and the owner abandons it (and it belongs to the finder). Or perhaps the kav per four square amot rule is because a kav is simply an insignificant amount, but two kavs are significant (and the owner does not renounce ownership)?
...The gemara answers: Teyku.

ASSUMPTIONS

A1 Assumption of Decision: \( f_i(k, a) \in \{0,1\} \) for all \( k, a \). (0 indicates despair.)
A2 Assumption of More-is-Better, Less-is-Worse:
\[
\begin{align*}
&f_i(k^*, a^*) = 0 \text{ implies } f_i(k, a) = 0 \text{ for all } k \leq k^* \text{ and } a \geq a^* \\
&f_i(k^*, a^*) = 1 \text{ implies } f_i(k, a) = 1 \text{ for all } k \geq k^* \text{ and } a \leq a^*.
\end{align*}
\]
A3 Assumption of Scale: The set of all pairs of \( k \) and \( a \) is \( R^+_2 = \{(k, a): a \geq 0, k \geq 0\} \).
A4 Midpoint Continuity: If \((k_1, a_1)\) and \((k_2, a_2)\) belong to \( D \) (or \( C \)), then the midpoint between these two points also belongs to \( D \) (or \( C \), respectively).
A5 Despair Over Negligible Fruit: All owners will despair over a de minimus amount of fruit regardless of the amount of land area: \( f_i(\varepsilon, a) = 0 \) for all \( a \).