

A Modern but Simple Approach to Teaching Friction

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Abstract- We present an experiment for measuring friction, using interleaved sheets of paper. Our results support the modern theory that friction is dependent on surface contact area (A). Also, our results support the classical theory of friction being proportional to the normal force (N), but this is only true over a limited range of N, where the “effective” area of contact increases in a manner proportional to N. Outside of this range, friction varies nonlinearly with N but linearly with A. Suggestions are made for extending the function of this experiment to measure the friction of other materials. Also, we present techniques to limit the nonlinear behavior of N and to eliminate several key sources of experimental error.

INTRODUCTION

The conventional way to teach friction is based on several classical concepts, which were formulated by Amontons and Coulomb [1,2]. These can be summarized by the equation

$$f = \mu N. \quad (1)$$

When an object slides on a second object, the force of friction (f) is directly proportional to the normal force (N) and the coefficient of friction (μ). The friction is not proportional to the contact area of the two objects. N is equal to the force component of the first object (its weight component plus other forces acting on it) acting inward and perpendicular to the second object. In simple terms: a truck has more friction than a car when both slide on the same road surface. The coefficient of friction represents the material properties of the two surfaces that are in contact: for example, an ice cube slides better on ice than rubber on concrete [3]. Friction is NOT dependent on the contact area between the two objects, at least not in the classical sense.

Using the classical concept of friction, friction can be measured by placing blocks of wood on a board. There may be different sheets of material attached to the block and board, and it is the friction between these materials that is measured. If we slowly incline the board with the horizontal plane (characterized by an angle θ), then when we just barely notice the block move, we can calculate the coefficient of static friction as [3,4] $\mu = \tan(\theta)$.

To find the coefficient of kinetic friction, the block (at rest) is tapped gently for some θ . If the block remains at rest, then θ is increased and the process repeated until the block moves down the plane after the tap. This method of measuring friction is often inaccurate due to inconsistencies both with the “smoothness” of the board/block system and with the human tapping the block.

More accuracy can be found using an airtrack [4]. By measuring the acceleration of a cart on an airtrack at an angle θ , a more accurate coefficient of friction can be derived. The only problem with this second approach is that the measured result is physically limited. In other words, we can only find the friction of the steel/air/aluminum interface, when we use an airtrack. (Note: air is actually a lubricant for the steel/aluminum interface.) With the first method, any two materials

can be attached to the wooden block/plane in order to measure the friction between them. Therefore, although the airtrack can measure friction more accurately, its results are of limited applicability. But both methods measure friction in a classical sense.

A third method for measuring friction is to place a block on a horizontal surface, attach a string, and run the string over a pulley to a mass that will use gravity to pull the block. The weight of this mass represents f , while the weight of the block is N . If f is too small, the block will not move. As f is increased, a point is reached where the block just begins to slide, and μ (for static friction) can be found from (1). Similar to the first case, the coefficients of various materials can be measured if they are attached to the block and sliding surface.

Modern Tribology teaches that friction is NOT directly a function of N . Rather, it is proportional to the effective contact area A_{eff} which represents the summation of all of the microscopic contacts between the molecules at the contact surface of the two objects [5,6,7,8]. A_{eff} is directly proportional (i) to the geometrical contact area A between the two objects as well as to (ii) the degree of surface-to-surface bonding. The first condition tells us that A_{eff} increases with an increase in contact area. The second condition says that A_{eff} increases with an increase in pressure (P) at the contact surface. With these new concepts, we can re-write (1) as

$$f = \mu AF(P) . \quad (2)$$

$F(P)$ is a function of the pressure between the two contacting surfaces. It may be linear or nonlinear. If we assume it is linear, then we can let $F(P)=P$, where we assume that friction is zero for no applied pressure, and this gives [9,10]

$$f = \mu AP. \quad (3)$$

If A is held constant, then f is proportional to P and $P = N/A$, so that friction is proportional to the normal force, as the classical approach states. If P is held constant (i.e. N/A is constant), then friction is proportional to A , as modern theory stresses. Let us consider 2 simple examples using (3).

EXAMPLE 1: Two blocks of the same mass (and therefore same N) slide on a surface. The second block has twice the contact area as the first. Hence, the average pressure for the first block is (N/A) , and for the second it is $(N/2A)$. Using (3), the friction of the first block is $f = \mu A(N/A) = \mu N$, and for the second block, $f = \mu(2A)(N/2A) = \mu N$. These results would lead one to conclude (wrongly) that the friction is proportional to N and not to A . This is the classical concept embodied in equation (1). As we will show later, these results are invalid if $F(P)$ is not linear in P .

EXAMPLE 2: If two blocks have contact areas A and $2A$ and if they have the same thickness (t) and the same weight density (N/At for the first block and $2N/2At$ for the second), then for the first block the friction is

$f = \mu A[t(N/At)] = \mu N$, and for the second it is $f = \mu(2A)[t(2N/2At)] = 2\mu N$. The friction doubles as the contact area doubles providing there is no change in the contact pressure (either N/A or $2N/2A$). The number of molecular bonds per area is the same, but the total number of bonds doubles for the larger area. Equation (3) correctly predicts this modern concept.

EXPERIMENTAL ANALYSIS

To study friction using equations (1) and (3), we took 2 clean pads of paper, with the cardboard backing removed. Each sheet was 21.5 cm wide (w) by 28.2 cm long. The notepads were interleaved. Each sheet of the first pad partially overlapped its corresponding sheet in the second pad: this is similar to a partial shuffle of a deck of cards. The overlap encompassed the full width (21.5 cm) of each sheet and a portion of the length (L , where L is clearly less than 28.2 cm). The free end of the first pad was clamped to the table, and it was not allowed to move. The free end of the second pad had a paper clamp attached, and this was attached to a string which ran over a pulley and down the side of the table to a mass hanger (50 gm). Weights were added to the mass hanger until the two notepads' "grip" slipped. The weight, in this case, is the force of the static friction. The weight of the string and pulley (about 15 gm) is included in our scattering error, which we will discuss later. The geometrical contact area is equal to w multiplied by the length (L), where L is typically between 5 and 15 cm. Since only the second pad moves, we choose the first pad to have $(n+1)$ sheets and the second pad to have n sheets. In this fashion, there are $2n$ paper/paper surfaces generating friction.

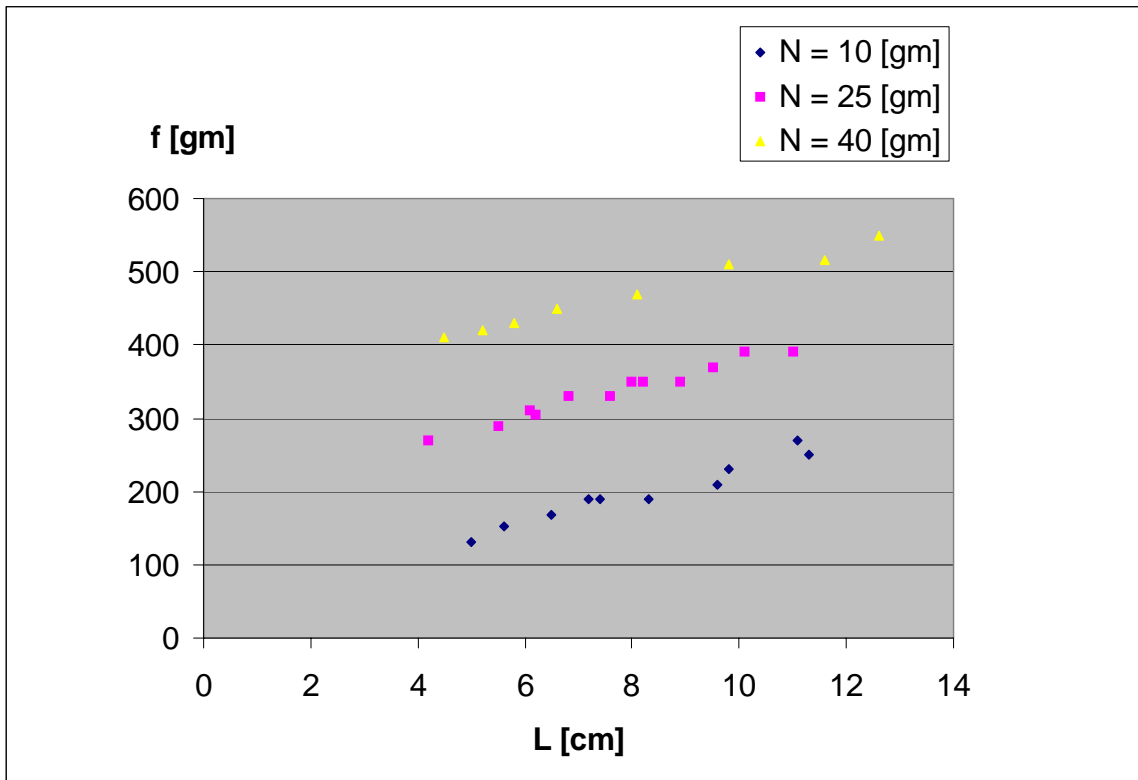


Figure 1: A plot of friction (in grams, since "g" is scaled out) versus the overlap L [cm] in two notepads of paper. For a given normal force ($N = 10, 25, \text{ or } 40$ [gm]), the plot of f vs. L is linear. Values for the y-intercept for each line $f(L=0)$ are extracted and plotted in figure 2. Values of slope for each line are extracted to find the coefficient of friction. See Table 1.

This method of measuring friction has the advantage that the effect of contact area is maximized. Over the interleaved portion of the pads, we placed a weight (N), which is a circular disc 3 cm in diameter. In order for L to be over the interleaved portion of the pads for all cases, L must be greater than 3 cm for all measurements. Typically we choose L greater than 5 cm. The effect of interleaved paper on the friction was to introduce a constant pressure-independent /area-

dependent term. The effect of the disc (N) was to introduce a pressure-dependent/area-independent term. These give

$$f = 2n\mu N + (2n+1) n\mu dL. \quad (4)$$

The first term in (4) is the classical equation (1) for friction, modified to characterize the $2n$ surfaces in this experiment. Equation (4) assumes that friction varies linearly with N and L . Our data indicates this to be true in all cases for L and in most cases for N . The variation becomes nonlinear for N large, i.e. $N \gg 0.5(2n+1)dL$. In the second term, d is the density of a single sheet of paper, given in mass per length. To obtain d , we measured the weight (actually mass in grams) of several hundred sheets of paper and divided by the number of sheets and also by 28.2 cm. We found $d = 0.118$ gm/cm. We also calculated the density s in units of mass per area, and $s = d/w$. We will use s later. For convenience, f and N were left in units of grams, though they can easily be converted to dynes or Newtons by multiplying by g (the acceleration due to gravity).

The second term in (4) is due to the weight of the paper alone. The factor $n(2n+1)$ comes about by counting the effect of the weight of the paper at each paper/paper contact: at the first surface, the normal force is caused by a single sheet of paper sitting above it (and fixed to the first pad, which does not move), at the second surface, two sheets of paper constitute the normal force, at the third surface, it is 3 sheets of paper, etc., and at the the last surface $2n$ sheets of paper are above the contact. The total friction due to the paper is the frictional force of one single sheet (μdL) multiplied by the sum of $1+2+3..+2n = n(2n+1)$.

We chose $n = 10$ and $N = 10, 15, 20, 25, 30, 35, 40, 45,$ and 50 gm. We interleaved the two pads for L approximately 12 to 14 cm. We measured L exactly, and then we added weights to the mass hanger until the pads started to pull apart. We pulled off several weights quickly and stabilized our pads. We measured a new L value, and then began adding weights until once again the pads began to pull apart. The weight at the point where the pads started to pull apart is the static friction, and this was plotted versus L for the various values of N used. See Figure 1. For clarity only, we show plots of f vs. L for $N = 10, 25,$ and 40 gm, although ALL data in this range showed the same behavior: f vs. L is a straight line variation with the scatter in data indicated. One can estimate the slope, intercept, error, and correlation (R^2) of each set of data points using a formal Least Squares analysis [11], or one can simply estimate these parameters from the graphs using a ruler to make approximate measurements. We found a correlation of our data to a linear fit of greater than 95%.

For each straight line plot in figure 1, the y-intercept and slope were found. The slope was constant for changing N (within the scattering error limits), and values of slope vs. N are listed in Table 1. This shows in a strong way that changing N has no effect on the change of friction vs. area, represented by the interleaved paper. Using the average slope listed in Table 1, we find $\mu = 0.590 \pm 0.070$.

Values for the y-intercept from figure 1, $f(L=0)$, are plotted in figure 2 vs. N . Equation (4) reduces to $f(L=0) = 2n\mu N$. Within the scattering error of our data, the y-intercept in figure 2 is 0, i.e. $f(L=0=N) = 0$. The slope is $9.01 = 2n\mu$, and this gives $\mu = 0.444$ with an error of 0.07, which corresponds well to the coefficient of friction reported in the literature [3,7]. Note also in figure 2 that $f(L=0)$ is linear in N up to $N = 40$ gm. Above this point, the variation is nonlinear. For $N = 40$, $\mu N = 17.5$ gm, and for maximum $L = 15$ cm, $Ld = 1.68$ gm. Therefore, if the first term in equation (4) is much larger than the second term (approximately 10 to 1), the friction depends on N in a nonlinear fashion.

Table 1

By finding the slope of the straight line F vs. L for a constant N and by using equation 4, we come up with coefficient of friction.

N [gm]	μ
10	0.593
15	0.636
20	0.593
25	0.597
30	0.650
35	0.566
40	0.551
45	0.565
50	0.571
Average	0.590

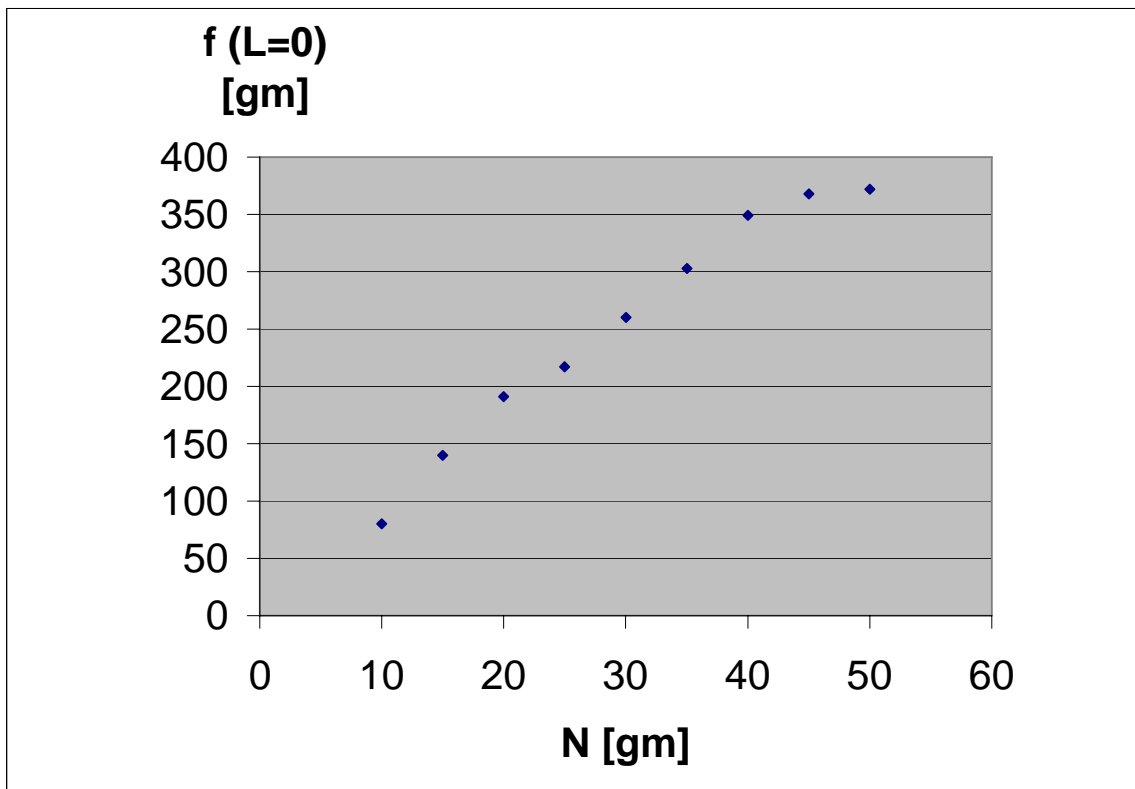


Figure 2: Values of the friction between two pads of paper for no overlap ($L=0$) of the two pads is plotted versus N . The graph shows a linear relationship, except for $N > 40$ [gm]. The slope of the linear portion of this line yields a coefficient of friction.

The area dependence of the friction on interleaved paper gives us $\mu = 0.590 \pm 0.070$. The changing normal force gives $\mu = 0.444 \pm 0.070$. The two values are approximately equal, with a 28% difference. The error (0.07) is based on a one-sigma scattering. Using a 2-sigma or 3 sigma scatter to define error would make the ranges of these two values of μ overlap in an unambiguous fashion. Thus, it is fair to say that μ is a constant in equation (4), providing we use a 2-sigma or 3-sigma error range to characterize μ . Nevertheless, there remains a 28% difference between values of the coefficient of friction found by varying the area and by varying the normal force.

To explore further, consider μ to be a function of N . We re-write equation (4) as

$$F = \mu(1) [2nN+n(2n+1)sa] + \mu(2) [n(2n+1) (dL-sa)], \quad (5)$$

where $\mu(1)$ is the coefficient of friction describing the paper surfaces immediately below the disc N , and $\mu(2)$ is the coefficient describing the paper surfaces surrounding the disc. For the experimental data reported, $dL \gg sa$, and $2N \gg (2n+1)sa$, so that equation (5) reduces to (4). Except that two different parameters ($\mu(1)$ and $\mu(2)$) are used to fit the variation of friction with area and normal force [9,10].

The dependence of μ on N is due to forces of adhesion between the surfaces in contact [12,13]. In general, if the surfaces are smooth, then the addition of a normal force produces an elastic deformation in the surface bonds. Friction varies linearly with N , and μ is constant. However, if N gets too large, a point is reached where plastic deformation occurs and

$$\text{Log}(\mu) = K\text{log}(N) + c, \quad (6)$$

where K and c are constants, and in most cases, $K=-1/3$. Our two values $\mu(1)$ and $\mu(2)$ are consistent with the behavior exhibited in (6). Since we do not have more than these two data points, we will not analyze (6) further. Rather, we merely cite the fact that N changes μ in a nonlinear way, and that increasing N tends to decrease μ .

There is one more factor affecting the quality of our data. We supplied a normal force (N) by using a disk. This produces an abrupt change in the boundary between paper with and without N above it. We performed this experiment again. Instead of a disk, we used a wooden ruler for N . This was placed over the entire width of the interleaved paper and extended an inch or two beyond. The mass of the ruler was recorded. Equal weights were attached to each end of the ruler in order to increase N .

The results of this new experiment were striking. Our data scatter decreased as our correlation (R^2) went from 95% to 97% (or more). The nonlinear behavior of N disappeared (up to $N = 235\text{gm}$). The difference between $\mu(1)$ and $\mu(2)$ was decreased significantly. We found

$$\mu(1) = 0.571 \pm 0.010,$$

and

$$\mu(2) = 0.621 \pm 0.018$$

The percent difference is 8%, a large improvement over the previous results.

CONCLUSION

We measured the frictional force between sheets of paper interleaved so as to maximize the effect of area. We found the friction to be linearly dependent on area in all of the cases studied. The friction is also linearly dependent on the external normal force N applied to the paper, but only for low values of N . The addition of the normal force strengthens the binding force between the molecular bonds that already exist at the paper/paper interfaces. However, if N becomes too large, the degree of binding saturates and does not increase appreciably with increasing N , or to put it another way, the value of the coefficient of friction decreases with increasing N . Nonlinear effects due to N can be eliminated by the careful choice of how N is applied.

We tried several other variations on this experiment. We changed n and w . The effect of changing w does not alter the behavior of the frictional forces already described nor does changing the number of sheets of paper used. However, if n is too large, the experiment can suffer from several sources of error. With $n=50$, we found the monotony of interleaving the sheets of paper tended to increase the human error. Also, with $n=40$ or 50 , weights of several kg were required to supply the frictional force. In many cases, this caused strings to break or clamps to come loose. By keeping n below 20, both of these sources of error were kept low, while maximizing the area dependence of friction.

One other problem is the build up of static electricity. After interleaving our notepads several times and pulling them apart, we noticed a build up of charge. When you lift up the top sheet of one notepad (before interleaving) the second sheet comes up as well. We avoided this problem by using new paper for each interleave. This required 200 (or more) sheets of paper for the full experiment. However, since the paper is not destroyed in the course of the experiment, there is no waste.

Other improvements on this experiment include efforts to study friction as a function of velocity and to study other material interfaces. A computerized test setup has been shown to be practical, accurate, and convenient [13], though it does depart from the simplicity of the experimental setup described here. Our present efforts are to keep the simplicity of the experiment intact while using computer control to reduce the effects of drudgery.

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Biography

Sarosh Patel received the B.E. degree in Electrical and Electronics Engineering with Distinction from the Faculty of Engineering Osmania University, India in 2002, and M.S. degrees in Electrical Engineering and Technology Management from the School of Engineering, University of Bridgeport (UB), in 2006. He is currently pursuing Ph.D. in Computer Engineering at U.B. He currently works as a Research Assistant at the Interdisciplinary RISC (Robotics and Intelligent Systems Control) Lab. He had been nominated for inclusion in 2005 & 2006 edition of Who’s Who Among Students in American Universities and has been elected to the Phi Kappa Phi honor society

Manan Joshi has received his MS degree in Electrical Engineering from University of Bridgeport in Dec 2006. Currently he is pursuing his PhD in Computer Science & Engineering at the University of Bridgeport. His research interests are in the field of Analog Electronics, Computer Networking and Wireless Communications.

Lawrence V. Hmurcik is Professor and Chairman of Electrical Engineering at the University of Bridgeport, Bridgeport, CT. He earned his Ph.D. in semiconductor devices at Clarkson University in 1980. He worked in Diamond Shamrock's research division for 3 years before joining the University of Bridgeport in 1983. Dr. Hmurcik has 50 publications and 5 grants. He is also a professional consultant with 240 case entries, including 14 appearances in Court and Legal Depositions. Dr. Hmurcik's interests have changed over the years: starting in Solar Cell technology in 1977, Dr. Hmurcik is currently pursuing work in Medical Electronics and Electric Safety.