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A Mathematical Model for Evaluating the Performance of Multicast Systems

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Abstract—The Internet is experiencing the demand of high-speed real-time applications, such as live streaming multimedia, videoconferencing, and multiparty games. IP multicast is an efficient transmission technique to support these applications. However, there are several architectural issues in this technique that hinder the development and the deployment of IP multicast such as a lack of an efficient multicast address allocation scheme. On the other hand, End System Multicasting (ESM) is a very promising application-layer scheme where all the multicast functionality is shifted to the end-users. Supporting high-speed real-time applications always demand a sound understanding of these schemes and the factors that might affect the end-user requirements. In this paper we attempt to propose both analytical and the mathematical models for characterizing the performance of IP multicast and ESM. Our proposed mathematical model can be used to design and implement a more efficient and robust ESM model for the future networks.

Index Terms— End-system multicasting, IP multicast, real time application, performance evaluation.

I. INTRODUCTION

There is an emerging class of Internet and Intranet multicast applications that are designed to facilitate the simultaneous delivery of information from a single or multiple senders to multiple receivers. Different approaches of multicasting have been suggested to improve the overall performance of networks especially the Internet. These approaches are: multiple unicast, IP multicast, and end-system multicast (ESM). All of these methods have some advantages and disadvantages but the last two approaches (IP multicast, and ESM) mentioned above have had more research effort in terms of performance evaluation of networks. Multiple unicast can be described as a service where one source sends the same copy of the message to multiple destinations. There is a one to one connection all the way from the source to the destination. In IP multicast, one source sends data to a specific group of receivers. In this case, a unique and special IP address is used, a class D address for the entire group. A tree rooted at the source is constructed and only one copy of the message is sent since the routers along the paths to the destinations performed the necessary replication functionalities. Finally, in an ESM

approach, host participating in an application session have the responsibility to forward information to other hosts depending on the role assigned by a central data and control server. In this case, the architecture adopted is similar to that of IP multicast with the difference that only IP unicast service is required. ESM uses an overlay structure, which is established on top of the traditional unicast services. The overlay has its meaning from the fact that the same link can have multiple unicast connections for multiple pair of edges.

II. RELATED WORK

Previous works about ESM [1], [2], have proposed experimental results and have been tested in very small group size scenarios including the Internet. There is other end-hosts multicast proposals like YOID [3], which is a set of protocols designed to build a new architecture for general content distribution. It considers three layers of protocols: an identification protocol, a transport protocol, and a tree formation protocol to construct optimal delivery trees. PBM [4] uses end-to-end delay bounds to reduce the delivery delays resulting from the well-known last mile bandwidth limitation providing a more scalable alternative. Multicast Service Nodes (MSN) [6] provides multicast services through a set of distributed multicast service nodes, which communicate with hosts and with each other using standard unicast mechanism. The MSNs act as proxies which forward and replicate data packets on behalf of the senders. Researchers usually refer to this generic advanced multicast model as Amcast (Overlay Multicast Network).

ALMI [5] approaches collaborative applications within a reduced number of group members. A fundamental challenge that ESM is facing is the fact of providing a method for nodes to self-organize into an *overlay network* that efficiently forwards multicast packets. These protocols primarily consist of two components: (i) a group management component, and (ii) an overlay optimization component. The first one ensures that the overlay remains connected in the face of dynamic group membership and failure of members. The second component ensures that the quality of the overlay remains good over time [1].

In this paper we are mainly concerned about end-to-end delay metrics for data-delivery process in Multiple unicast, IP multicast and ESM. From the proposed mathematical model and the simulation results, we can observe that there is no significant difference when comparing ESM to IP multicast for a small size of network. Besides, ESM represents a low-cost solution to multicast service demand because there is no need to pay for additional support from ISP or other network service. However, it is still experiencing some limitations in scalability, latency and bandwidth management.

III. THEORETICAL ANALYSIS

In this section, we will theoretically analyze the problems of different level of multicasting, which hinder their performance with respect to the bandwidth utilization and latency.

A. Multiple Unicast System

In the unicast IP network, the host acting as the source transmits a copy of the message to each destination host as shown in Fig. 1. No special configuration is needed either in the source or in the core network. The intermediate routers will have to carry all these messages to the proper destinations. The chains of protocol entities that take care of the transmission process also use processing capacity on the host for each transmission. In addition, the transmission time is typically increased with some magnitude and it will affect the global end-to-end delay. These are the reasons to consider a multiple unicast service an unpractical approach to implement on the network.

B. IP Multicast

IP multicast is a service where one source sends data to a group of receivers each of them containing a class D address as membership identification. In IP multicast, a packet is sent only once by the source. Routers along the route take care of the duplication process. The IP-multicast capable version of the network shown in Fig. 2 consists of network with native

multicast support. The traditional process includes the construction of a source-rooted tree together with the members of the multicast group. Since only one copy of the message is required, we can say that a minimum bandwidth effort is being used for the transmission of the message to all group members connected in the network. The IP-multicast transmission takes the same bandwidth on source host's network as a single copy, regardless of how many clients are members of the destination host group in the Internet.

However, the main disadvantage of IP multicast is the need of commercial routers supporting multicast protocol. In theory, almost all routers support multicast but in practice this is not the case. Investors still think that there is not enough multicast application demand and that multicast traffic could take their routers down due to congestion problems.

Several approaches to multicast delivery in the network have been proposed which make some improvements or simplifications in some aspects, but they do not improve upon traditional IP multicast in terms of deployment hurdles. A major obstacle for deployment of multicast is the necessity to bridge from/to the closest multicast router to/from the end-systems. Existing IP multicast proposals [1, 7] embed an assumption of universal deployment, as all routers are assumed to be multicast capable. The lack of ubiquitous multicast support limits the deployment of multicast applications, which in turn reduces the incentive for network operators to enable multicast. Therefore, from the above discussion one can expect that we need another multicast alternative in which network routers have not to do all of the work; instead each of the host will equally contribute in the overall multicast process of the messages.

C. End-system Multicast (ESM)

ESM is a very promising application layer solution where all the multicast functionality is shifted to the end users as shown in Fig. 3. There is one central control server and one central data server residing in the same root source as shown in Fig. 3. In ESM, host participating in an application session can

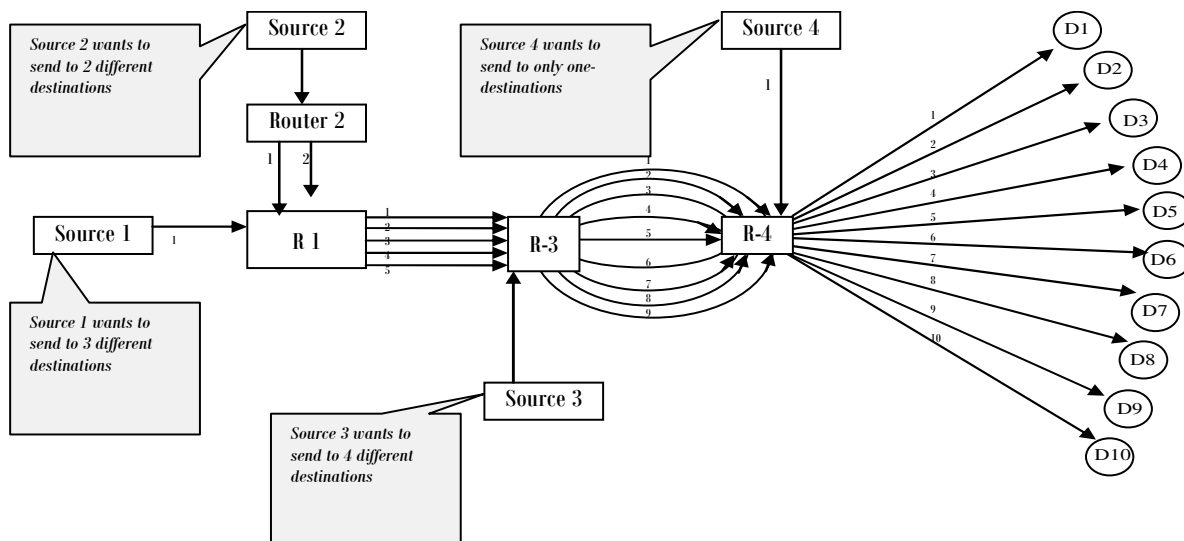


Fig. 1 Example of multiple unicast

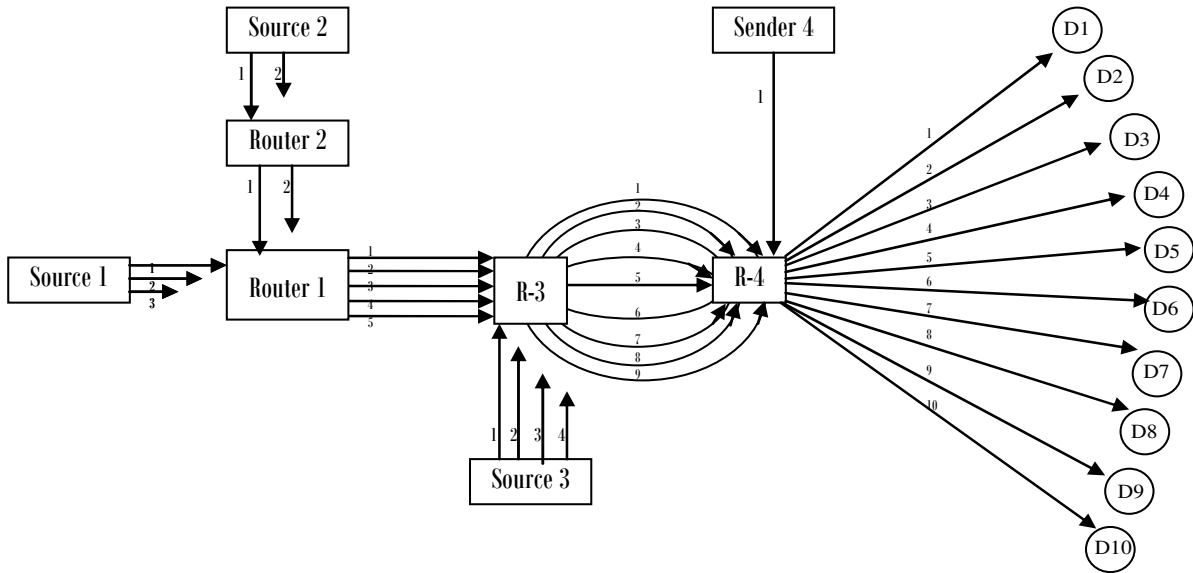


Fig. 2 Example of IP multicast

have the responsibility to forward information to other hosts. Here, end users who participate in the multicast group communicate through an *overlay* structure. However, doing multicasting at end-hosts incurs in some performance penalties. Generally, end-hosts do not handle routing information as routers do. In addition, the limitation in bandwidth and the fact that the message needs to be forwarded from host-to-host using unicast connection, and consequently incrementing the end-to-end delay of the transmission process, contribute to the price to pay for this new approach. These reasons make ESM less efficient than IP multicast. The structure of the ESM is an overlay in a sense that each of the paths between the end systems corresponds to a unicast path. The end receivers could play the role of parent or children nodes. The parent nodes perform the membership and replication process. The children nodes are receivers who are getting data directly from the parent nodes. Any receiver can play the role of parent to forward data to its children. Each client has two connections: a control connection and a data connection.

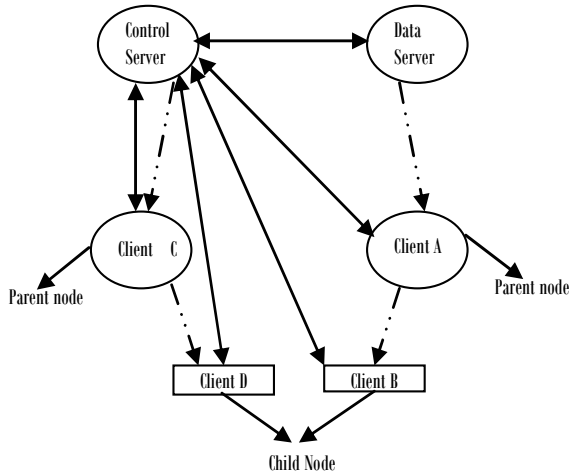


Fig. 3 Example of End-System Multicast

IV. PROPOSED MATHEMATICAL MODEL FOR MULTICASTING

Let G is an irregular graph that represents a network with a set of N vertices and M edges such as: $G = \{N, M\}$. Let L is a direct communication link between a single pair of source (s) and destination (d) where both source and destination belong to N such as: $\{s, d\} \in N$. In addition, each packet transmitted between source (s) and destination (d) must traverse one or more communication links in order to reach the final destination. Let the value of $D(L)$ denotes packet-delay that is associated with each direct communication link. Therefore, each transmitted packet will typically experience a delay of $D(L)$ on a particular link. In connection less communication such as IP network, there might be multiple routes exist between a pair of source and destination. As a result, each packet might follow a different route in order to reach the final destination where each route requires traversing of one or more communication links (L). A single route between a pair of source and destination can be defined as: $R\{s, d\}$ where $\{s, d\} \in N$

A. Mathematical Model for a Unicast System

In unicast, a packet is sent from one point (source) to another point (destination). As mentioned earlier, when packet transmit from one source (s) to a specified destination (d), there exist multiple routes where each route can have multiple links. This implies that the packet-delay for unicast is entirely dependent on the number of links a packet needs to traverse in order to reach the final destination system. Based on the above argument, one can define the packet delay such as: $D(R) = D(L_1) + D(L_2) + \dots + D(L_n)$ where n is the maximum number of links that need to be traversed on route R between s and d . The delay can be generalized for one particular route (R) that exist between source (s) and destination (d) such as:

$$D(R) = \sum_{i=1}^n D(L_i) \quad (1)$$

$$\text{where } \sum_{i=1}^n (L_i) = L_1 + L_2 + \dots + L_n \in M$$

Equation (1) can be further expressed as:

$$\text{Delay} = D_{(s-d)} = \sum_{L \in R(s-d)} D(L) \quad (2)$$

where $L \in R(s-d)$ represents the value of the total delay associated with the route R between source s and destination d . Based on (1) and (2), one can also simply derive a mathematical expression for estimating an average-delay (denoted by AD) which each packet may typically experience if it traverses one of the available routes. The mathematical expression for an estimated AD is as follows:

$$AD_{(s-d)} \hat{=} \left(\sum_{i=1}^y D(R_i) / y \right) \quad (3)$$

where y represents the maximum number of possible routes between source s and destination d .

In addition to the average delay, one can also chose the optimal route with respect to the minimum delay that each packet may experience when traverse from one particular source to a destination such as:

$$OD_{(s-d)} = \text{Min} \left| \sum_{i=1}^y D(R_i) \right| \quad (4)$$

The above derivations can be further extended for the multiple unicast system where a single source (s) can transfer a packet simultaneously to multiple destinations. This hypothesis leads us to the following argument: multiple routes can be established between the source (s) and each destination system. The following mathematical expression can be used to estimate the total delay that the entire packet transmission will experience in a multiple unicast system:

$$D_{\langle s \xrightarrow{\text{multiple}} (d_1, d_2, \dots, d_y) \rangle} = \sum_{i=1}^y D(R_i) \quad (5)$$

where y in (5) is the maximum available unicast routes between a particular source (s) and multiple destinations.

Although, in multiple unicast system, a single packet can be transmitted from one source to multiple destinations, the transmitted packet may follow a different route in order to reach the appropriate destination. Consequently, each packet transmission may yield a different delay depending on the number of links the packet needs to traverse on the chosen unicast route. This leads us to the following mathematical expression for an average delay:

$$AD_{(s-d)} \hat{=} \left(\sum_{i=1}^y \frac{D(R_i)}{y} \right) \text{ where } D(R_i) = \sum_{i=1}^n D(L_i) \quad (6)$$

where y represents the maximum number of unicast routes between a source (s) and multiple destinations and n represents the maximum number of links a unicast route has.

B. Mathematical Model for Multicast System

In IP multicast system, a single source (s) sends a packet to a group that consists of multiple destination systems. In addition, a packet is sent only once by the source system where as the intermediate routers along the route perform replications with respect to the number of destinations a group has. Let M_G denotes a multicast group that consists of one or more destination systems whereas Z represents the size of the group such as $Z = |M_G|$. In an IP multicast system, all multicast groups (M_G) can be typically organized in a spanning tree. We consider a spanning tree rooted at the multicast source (s) consisting of one of the multicast groups (M_G) that has a size of Z . The spanning tree can then be expressed as: $T = (N_T, M_T)$ where the numbers of destinations in one multicast group (M_G) belong to the total number of nodes present in the network such as: $M_G \in M$. Also, Based on the above discussion, we can give the following hypothesis: The total delay (D) experienced by multicast packets when transmitted from a root node (s) to a multicast group (M_G) can be defined as a sum of total delay experienced by each link of a spanning tree from the root nodes (s) to all destinations ($d \in M_G$) and the delay experienced by each link of an intermediate routers. Thus, this leads us to the following expression for total delay (D) experienced by multicast packets transmitted from root node (s) to a destination node (d):

$$D_{(s-M_G)} = \sum_{i=1}^Z D(L_i) + \sum_{i=1}^n D(L_i) \quad (7)$$

where Z is the number of destination systems in one multicast group of a spanning tree (T) where n represents the total number of links a route has.

The first term of (7) yields the total delay associated with the number of links with in a spanning tree when a packet is transmitted from a root node (source) to all the leaf and non-leaf nodes. The second term of (7) provides a total delay that a packet may experience when transmitted along a certain route. Equation (7) can be further generalized for one of the specific destinations (d) within a multicast group such as $d \in M_G$, if we assume that we have a route within a spanning tree (T) from multicast source (s) to a specific destination (d) such as $R_T(s, d)$, then the multicast packets transmitted from a source node to a destination experience a total delay of:

$$D_{\langle s \longrightarrow (d \in M_G) \rangle} = \sum_{L_{n,Z} \in R_T(s,d)} D(L_{n,Z}) \quad (8)$$

where $L_{n,Z}$ represents the total number of links (i.e., $Z \in R_T$) that a packet needs to traverse in order to reach the specific destination d along a path of R_T with in the tree T as well as the number of links from source s to a multicast group M_G .

C. Mathematical Model for an End- System Multicast

Because of the limitations in IP multicast, researchers have explored an alternative architecture named ESM, which is built on top of the unicast services with multicast functionalities. In ESM, one of the end-system nodes (s) participating in an application session can have the responsibility to forward information to other hosts. Here, end users that participate in the ESM group communicate through an *overlay* structure. An

ESM group can have at most N end-system nodes where we focus on one of the end-system nodes (s) that multicast information to the other participating nodes of a multicast end-system group. From the source host point of view, this ESM group can be considered a group of destination systems. For the sake of mathematical model, lets ESM_G denotes an ESM group that consists of one or more end-system destination where as X represents the size of the group such as $X = |ESM_G|$. Based on the derived expression of unicast in the previous sections, these unicast links can not exceed to M such as $m_1, m_2, \dots, m_y \in M$ where one of the edges provides a unicast connection between two end-system nodes such as:

$$\{m \in M\} \xrightarrow{\text{unicast-link}} \{n_1, n_2\} \subset \{s, N\} \quad (9)$$

An overlay network consists of a set of N end-system nodes connecting though M number of edges where one of the end-system is designated as source host (s) such as: $G = \{s, N, M\}$. This also shows that an ESM is built on top of the unicast services using a multicast overlay network that can be organized in a spanning tree such as $T = (N_T, M_T)$ rooted as an ESM source (s) where the numbers of destinations in one multicast group (ESM_G) belong to the total number of nodes present in the network such as: $ESM_G \in M$. The end receivers in a multicast tree could be a parent or a child node depending on the location of the node. In a multicast spanning tree (T), all the non-lead nodes can be both parent and child at the same time where as all the leaf nodes are considered to be the child nodes. Based on the above argument, one can say that a multicast packet originated from the root (s) of a spanning tree (T) need to traverse typically two links; source to non-leaf node (P_n, C_n) and a non-leaf node to a leaf node (C_n). Lets R_T (s , non-leaf node) represents a route between a source node (s) and non-leaf nodes that could be parent or child nodes such as:

$$R_T = \{P_n \vee C_n \in ESM_G\} \text{ where } P_n, C_n \in \{s \cup N\} \quad (10)$$

where, $R_T(P_n, C_n)$ represents a route from a parent node to a child node such as: $R_T = \{P_n, C_n\} \in ESM_G$.

Equations (9) and (10) lead us to the following expression for computing the total delay involve in transmitting a multicast packet from a source node to one or more parent nodes (i.e., the delay associated with the first link of transmission):

$$D_{\langle s \xrightarrow{\text{multiple-unicast}} (P_n \vee C_n) \in \{s \cup N\} \rangle} = \sum_{i=1}^y D\left(R_{i(s \rightarrow P_n, C_n)}\right) \quad (11)$$

where $D\left(R_{ii(s \rightarrow P_n, C_n)}\right) \triangleq \sum_{i=1}^n D(L_i)$

In (11), y is the maximum unicast routes between a source (s) and one or more non-leaf nodes and n represents the maximum number of links a unicast route can have. Similarly, the total delay experience by a multicast packet transmitted

from a parent node to a child node can be approximated as follows:

$$D_{\langle P_n \xrightarrow{\text{multiple-unicast}} (C_n) \rangle} = \sum_{i=1}^y D\left(R_{i(P_n, C_n)}\right) \quad (12)$$

where $D\left(R_{i(P_n, C_n)}\right) \triangleq \sum_{i=1}^n D(L_i)$

By combining (11) and (12), the total delay experience by a multicast packet that transmitted from a source node (s) to a child node (C_n) can be approximated as:

$$D_{\langle s \xrightarrow{\text{multiple-unicast}} (C_n) \in \{s, P_n \cup N\} \rangle} = \sum_{i=1}^n D\left(L_{i(s \rightarrow P_n, C_n)}\right) + \sum_{i=1}^n D\left(L_{i(P_n, C_n)}\right) \quad (13)$$

V. PERFORMANCE ANALYSIS

A. System Model

Simulations are performed using OPNET to examine the performance of Multiple unicast, IP multicast, and ESM schemes. Figure 4 shows an OPNET model for the Multiple unicast, IP multicast and ESM simulations. The OPNET simulation has run for a period of 900 seconds for all three scenarios where we collect the simulated data typically after each 300 seconds. For all scenarios, we have setup one sender node that transmits video conferencing data at the rate of 10 frames/s using 2,500-stream packet size to one or more potential receivers via a link that operates at 100 Mbps. In addition to these 100 Mbps links, we use separate DS3 links for the core network (Internet). The same traffic pattern is assumed for all scenarios.

It should be noted in Fig. 4 that we use four backbone routers that connect multiple subnets to represent Bay Networks concentrator topology using ATM – Ethernet FDDI technology. In order to generate consistent simulation results, we use the same topology for the first two scenarios with some minor exceptions. For Multiple unicast, we disable the multicast capabilities of backbone routers where as for the IP multicast this restriction does not impose. Finally, in order to examine the behavior of the ESM, we use an OPNET Custom Application tool that generates the overlay links and the source root.

B. Experimental Verifications

For the Multiple unicast scenario, video conferencing data is being sent by the root sender at the rate of 25 K-bytes per second. This implies that a total of three copies traveling which result in 75 K-bytes per second of total traffic. The last mile bandwidth limitation typically provides the most important delay impact. OPNET collected all the delays for all the receivers and calculated the average. The packet end-to-end delay for Multiple unicast was 0.0202 seconds. For the IP multicast approach only *one copy* of the packet is generated at the root source. For this reason, the total video-conferencing

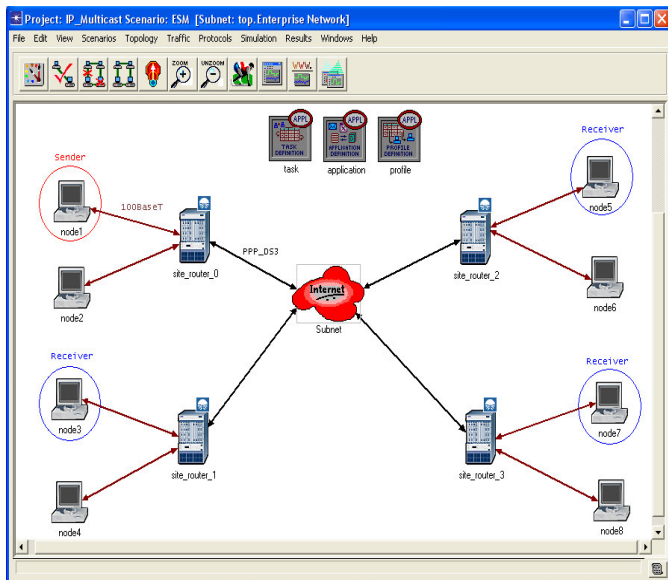


Fig. 4 OPNET model for Multiple unicast, IP multicast and End-System Multicast videoconferencing transmissions.

traffic sent and received is only 25,000 bytes/s. Thus, a better performance in the average packet end-to-end delay can be observed. This is approximately 0.0171 seconds. Finally, after performing ESM simulations, we obtain an average end-to-end delay packet of 0.0177 seconds.

It can be seen in Fig. 5 that ESM packets transmission provides comparatively good performance than the Unicast but not as good as the IP multicast. The reasons are the RDP (Relative Delay Penalty or the ratio of the delay between the sender and the receiver) [2] and the LS (Link Stress or the number of identical copies of a packet carried by a physical link) experienced by each network schemes. Even though, a Unicast scheme provides comparatively low RDP, the value of LS is not optimal. On the other hand, IP multicast performs with a little bit higher RDP but it gets a better LS. ESM has the inconvenience of RDP higher than IP multicast due to the fact that for a second receiver, there is an increasing delivery-delay because of the end-user replication (the second user has to wait for the data sent by its father node or sub-server). This is the penalty that ESM has to pay. One possible solution would be the design of a robust multicast protocol to optimize the delivery of data for the final users. Note that the additional delay could be reduced if we optimize the bandwidth utilization in the potential parent nodes. This is not a simple task because it requires a smart protocol to recognize bandwidth limitations in potential parent nodes and to establish an algorithm to limit the number of children nodes for these parent nodes.

VI. CONCLUSION

In this paper, we presented a complete mathematical model that can be used to evaluate the performance of multicast systems. Specifically, the proposed mathematical model can be used to compare the performance of the ESM, the IP multicast and the multiple unicast topologies. We concluded that ESM is a promising alternative for the next generation networks. For

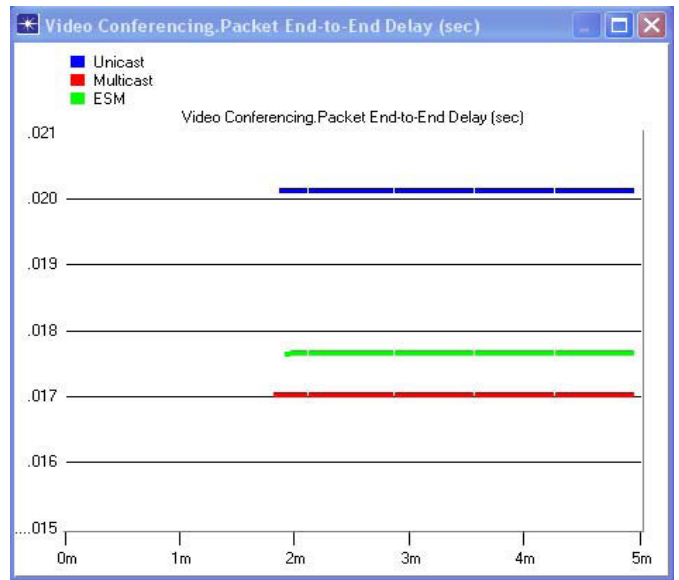


Fig. 5 Average end-to-end packet delays for Multiple unicast, IP multicast and ESM simulations.

the future work, it will be interesting to extend and implement the proposed mathematical model to measure the bandwidth consumption and the overall data throughput per system.

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